**A Decision Analysis Perspective on Multiple Response Robust Optimization**

THESIS

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# A DECISION ANALYSIS PERSPECTIVE ON MULTIPLE RESPONSE ROBUST OPTIMIZATION

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## Abstract


Decisions in which multiple objectives must be optimized simultaneously occur frequently in government, military, and industrial settings. One method a decision maker (e.g., a design engineer) may use to assist in multiple response optimization situations is the application of a desirability function. The decision maker specifies the desirability function parameters so as to express his or her own preferences with respect to the objectives under consideration. An informed specification of the parameters is essential so that the desirability function accurately describes the decision maker's value trade-offs and risk preference. Misapplication of the desirability function may result in the selection of an optimal policy that is inconsistent with the stated preferences. This thesis examines the desirability function from a decision analysis perspective. In particular, utility transversality provides the basis for an analysis of the implicit value trade-off and risk attitude assumptions attendant to the desirability function.

A limitation of the desirability function is its failure to explicitly account for response variability. A robust solution accounts for not only the expected response, but the variance as well. Assessing a utility function over desirability as a means to describe the decision maker's risk preference produces a robust operating solution that is consistent with those preferences. This thesis examines robustness as it applies to the desirability function, using a decision analysis perspective. In particular, a robust manufacturing solution is identified for a wire-bonding process, seen often in the quality and reliability engineering design literature. An exponential utility function over desirability is applied to regression equations developed from a Box-Behnken design. Monte Carlo simulation enables specification of the robust solution.

Using decision analysis methods, this methodology is applied to a practical problem currently facing the Air Force Research Laboratory (AFRL). Contributing to AFRL's Robust Decision Making Strategic Technology Team program, this thesis examines robustness in the context of national policy-making in country stability situation. Different levels of diplomatic, informational, military, and economic (DIME) instruments of national policy are investigated to examine how they affect the political, military, economic, social, infrastructure, and information (PMESII) systems of a nation. AFRL's National Operational Environment Model (NOEM) serves as the basis for the analysis of a scenario involving the Democratic Republic of Congo. A D-optimal design of experiments enables identification of a robust national policy. Employment of a multiattribute utility function that satisfies the axioms of expected utility theory ensures that the policy is consistent with the decision maker's stated preferences.

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Jonathan S. Findley

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A DECISION ANALYSIS PERSPECTIVE ON MULTIPLE RESPONSE ROBUST OPTIMIZATION

# I. Introduction

Often, a decision must be made that optimizes a single objective, such as a company desiring to maximize profit. Other times, decisions must be made to simultaneously optimize a set of multiple objectives. Such decisions can occur within any organization.

Examples of multiple objective decisions in government include selecting the proper regional road network that balances the environmental, social and economic impacts of the region's municipalities within a fixed budget [7]. Another example is selecting the best location for a new airport. The competing objectives in this case are minimizing construction costs, while also minimizing transit time for travelers, maximizing safety and minimizing noise pollution [21].

Examples in military settings include designing a military aircraft where maximum range and maximum speed are two competing objectives with maximum payload capacity [25]. Another example is developing a stabilization, security, transition, and reconstruction operation (SSTRO) where the nation's positive political, military, economic, and social indicators are desired [5].

Examples in industry include gum extraction from plant seeds where maximum extraction yield, viscosity, hue, and emulsion stability needs to be balanced with minimum protein content [28]. Another example is machining parts where the balance of metal removal rate and surface roughness needs to be optimized [35]. One's personal life also holds examples of such decisions such as buying a truck where one desires maximum power with maximum fuel economy.

Often, objectives conflict so that selecting an alternative resulting in the simultaneous optimum for each objective is infeasible. Consider the truck buyer desiring increased power in a new truck with increased fuel economy. Nearly always, the truck with the most power will not also have the highest fuel economy. This "ideal" truck does not exist. In these cases, a compromise solution to balance these competing objectives must be found. This correct balance is based in large part on the application and objectives in question as well as the preferences of the decision makers in charge. Although many organizations have a group of people making these types of decisions, this thesis assumes a single decision maker with no loss of generality. Several areas of study exist to assist the decision maker with such decisions.

Decision analysis is a combination of mathematical and logical methods used to assist a decision maker in choosing the appropriate decision in an uncertain world. Using the axioms of utility theory, mathematical models can be created that best describe the preferences of the decision maker [18]. These preferences include the decision maker's trade-offs between different value measures and the decision maker's probabilistic preference between different outcomes [18]. Decision analysis can apply to both single attribute situations as well as multiple attribute situations.

Some decision analysis practitioners prefer to treat multiple attribute applications in a way that examines each attribute separately in terms of the decision maker's preferences for that attribute. Marginal utility functions are developed for each attribute; these separate functions are then combined into a single multiattribute utility function based on certain assumptions made about preferential independence, utility independence and additive independence amongst the attributes [23].

Other decision analysis practitioners prefer to formulate a value function directly considering the deterministic trade-offs between the multiple attributes. A utility function is then assessed for this value measure [1]. Matheson and Abbas propose the idea of utility transversality in a utility function assigned over a value function [31]. This concept relates the risk aversion functions of the individual attributes to the value trade-off functions between these attributes. Abbas also relates a decision maker's risk aversion over value to the decision maker's multiattribute risk aversion [2].

Often, a decision maker is interested in adjusting a set of controllable inputs to optimize a set of outputs. Response Surface Methodology (RSM) is a set of statistical and mathematical methods used to develop or improve processes [32]. Used widely in industry, most RSM applications optimize a response variable which is a function of one or more input variables. These functions can be known exactly such as through a chemical or engineering process. Other times, the underlying function is not known and is estimated using various statistical methods [32].

In many cases, more than one response variable is important to a process. One example is a machining process with machining parameters as input variables and two response variables: removal rate and surface roughness. The optimum decision might be the proper combination of machining parameters that simultaneously maximizes removal rate and minimizes surface roughness [35].

Numerous multiple response optimization models exist in current RSM literature [15, 9, 13, 24, 45]. Harrington introduces one such model [15]. Harrington's desirability function transforms each response to an individual desirability level between zero and one. The optimum strategy is then the one which produces the highest geometric mean of the individual desirability levels [15].

Derringer and Suich modified the form of the individual desirability function to be more flexible [13]. They change the individual desirability function to provide the decision maker more control over how quickly desirability moves from zero to one as the response moves from its worst value to its target value.

In employing the desirability function, care must be taken by the analyst to choose parameters consistent with the decision maker's own preferences. The desirability function exhibits implicit and explicit assumptions regarding risk attitude, value trade-offs, and attribute independence. Potential problems may result from using a multiple response optimization model without fully understanding these assumptions. Kros and Mastrangelo introduce the idea of applying utility theory to examine the assumptions and preferences underlying desirability functions [29]. They attempt to compare and contrast assumptions regarding risk preferences, trade-offs, and relationships between the multiple attributes inherent in Derringer and Suich's desirability function [29].

This thesis examines the desirability function from a decision analysis perspective. This analysis provides knowledge about how best to employ the desirability function in a manner consistent with the decision maker's value trade-offs and risk attitude.

In the current climate of budgetary constraints across industrial and governmental organizations, proper analysis of a decision situation is vital to maximizing the limited resources available. When confronted with a situation where multiple objectives need to be considered, the analyst needs to understand the assumptions inherent to the model chosen so that it is consistent with the decision maker's preferences and the proper solution for the organization can be found.

The rest of this thesis is organized as follows. Chapter 2 presents a review of relevant literature. Chapter 3 develops the methodology used to examine the desirability function and the methodology used to find a robust optimum solution from a decision analysis perspective. Chapter 4 presents the analysis of a robust optimization solution in a wire bonding process experiment in the semiconductor industry using various assumptions about risk attitude and value trade-offs. Chapter 5 presents the analysis of a robust optimization solution in a nation-building (i.e., SSTRO) example. Chapter 6 offers significant findings, recommendations, and suggestions for future work.

# II. Review of Related Literature

## 2.1 Organization

This chapter reviews the related literature applicable to the thesis. Section 2.2 gives an overview of decision analysis including risk preferences and modeling uncertainty in cases of single attributes and multiple attributes.

Section 2.3 discusses Response Surface Methodology and how the desirability function is used to find an optimum setting in a multiple response situation. The idea of a robust optimum point in a noisy environment is also discussed.

Section 2.4 discusses stability operations and their place in the United States security plan are then discussed. The Air Force Research Lab models such operations with their National Operational Environment Model.

## 2.2 Decision Analysis

In a system of unknowns such as a business venture where future profits and product demand growth can only be estimated, decision analysis is an excellent tool to balance the factors, both certain and uncertain, that apply to a given decision situation. Ronald Howard describes decision analysis as a cycle encompassing deterministic, probabilistic, and informational phases to settle on a logically best decision [18]. The deterministic phase establishes certain relationships between the variables within the problem. The probabilistic phase introduces the uncertainties and risk preference of the decision maker. The informational phase determines the value of gathering more information. If more information is to be gathered, the cycle is repeated until no new information is deemed necessary by the decision maker [18].

The uncertainty involved in a business investment venture is not something easily measured. Howard describes two types of probabilities: objective and subjective. An objective probability is one that is measured after several instances of the uncertain event occur. For example, finding after 1,000 coin flips that the coin came up heads approximately 500 times could lead one to objectively assign a 50% probability to that coin coming up heads on the next coin flip. Alternatively, a subjective probability is one which is assigned based on individual knowledge about the nature of the uncertain event. For example, assigning a 50% probability to a coin one has never seen flipped before based on the fact the coin appears to be 'fair' is subjective [18]. In most cases, a particular business venture cannot be tried several times to see how likely it is to be successful. Only a subjective probability can be assigned to its success based on analytical knowledge of the nature of this and similar ventures.

### 2.2.1 Utility Theory.

Howard describes utility theory as encompassing five axioms concerning the idea of lotteries [18]. A lottery is a set of outcomes in which exactly one occurs. The first axiom requires transitivity in preference. If three alternatives, A, B, and C, are available and if an individual prefers A to B and prefers B to C, this individual must prefer A to C.

The second axiom calls for a probabilistic preference. Consider one who prefers A to B to C. Then, a preference probability, $p$, must exist where this person is indifferent to accepting B for certain or accepting a lottery which produces A with probability $p$ or produces C with probability $(1 - p)$. When a particular value of $p$ is found, B becomes the certain equivalent (CE) of the lottery between A and C. Figure 1 shows how this lottery and its CE are usually depicted in decision analysis literature.

The third axiom involves substitution. The CE of a lottery can be exchanged or substituted for the lottery itself in any situation without any changes of preference for the decision maker.

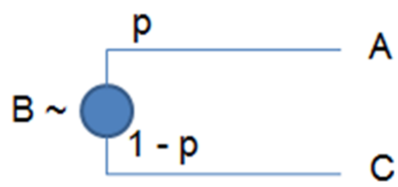


**Figure 1. B is the certain equivalent of the lottery between A and C**

The fourth axiom states the acceptance of probability as the means to describe uncertainty and also expresses indifference in the ordering of lotteries in the decision at hand. It states that multiple levels of lotteries may be replaced by a single lottery with the prizes and associated probabilities calculated by the laws of probability. In essence, this axiom states that the lottery itself holds no intrinsic value to the decision maker.

The fifth axiom states if a decision maker is faced with two lotteries, each with outcomes of A or B, and the decision maker prefers A to B, the decision maker must prefer the lottery that yields A with the higher probability.

#### 2.2.2 Risk Preference.

Any practical decision opportunity which involves uncertainty must take into account the decision maker's preference towards risk. Howard points out that few people would be willing to give up next year's salary for a 50% chance of doubling it versus receiving no salary for the year although this represents a fair proposition [18]. A person indifferent to the preceding deal is considered to be risk neutral. Howard suggests that most people and organizations are risk-averse. A risk-averse individual's CE for a given lottery is less than the expected value of the outcome of that lottery [18].

### 2.2.3 Utility Function.

One way to calculate a decision maker's risk preference is to find the preference probabilities for all feasible alternatives within the current decision space. Depending on the situation and the number of alternatives, this can become quite cumbersome. The utility function encodes the decision maker's risk preference. This function associates an outcome from a lottery or other uncertain situation with a utility value. Moreover, the decision maker's utility of a lottery is the expected value of the u-values of the lottery's outcomes. From the example in Figure 1, the decision maker's utility for the lottery would be $u_1 = p * U(A) + (1 - p) * U(C)$. If the decision maker prefers one lottery to another, the preferred lottery's utility will be higher than the other.

Howard also points out that although these u-values can model the decision maker's preferences among various alternatives or lotteries, the actual magnitude of a utility means nothing on its own [18]. Comparing utilities cannot be used to show how strongly one alternative is preferred to another. It can only be used for ranking purposes [18].

Arrow and Pratt [6, 37] introduce the risk aversion function,

$$\gamma(y) = -\frac{u''(y)}{u'(y)}. \tag{2.1}$$

They show the negative ratio of a utility function's second derivative over its first derivative correctly measures the local risk aversion at any given point along a utility function. This measure assumes that the utility function in question is monotonically increasing and is twice differentiable. If $\gamma(y) = 0$, the utility function is describing risk neutral behavior. If $\gamma(y) > 0$, it is describing risk averse behavior and if $\gamma(y) < 0$, it is describing risk seeking behavior [37].

### 2.2.4 Modeling Uncertainty.

Decision analysts employ several methods to model uncertainty. Two common methods are using a discretized approximation of the continuous distribution or using a Monte Carlo simulation over the distribution [11]. A discretized approximation simplifies the calculations involved with using a continuous function. When the continuous probability function is unknown, it can be estimated by using known data to estimate the parameters of a Beta distribution.

The Beta distribution is a flexible continuous probability distribution given over a set bounded range [3]. Equation 2.2 shows the density function.

$$Beta(\alpha, \beta, a, b, x) = \frac{(x-a)^{\alpha-1}(b-x)^{\beta-1}}{\int_a^b (x-a)^{\alpha-1}(b-x)^{\beta-1}dx} \tag{2.2}$$

Where $a$ and $b$ are the lower and upper bounds of the domain, and $\alpha$ and $\beta$ are the two parameters of the Beta distribution [3].

Abbas et al. [3] describe assessing the data into fractiles and then using Matlab's *fminsearch* function to estimate the two parameters $(\hat{\alpha}, \hat{\beta})$ by minimizing the squared errors from the value, $x$, from the inverse of its related probability $p$. Equation 2.3 shows the relevant expression,

$$\min_{\hat{\alpha},\hat{\beta}} \sum_{i=1}^{n} (X_i - \hat{X}_i)^2, \tag{2.3}$$

where $\hat{X}_i = BetaInverse(p_i, \hat{\alpha}, \hat{\beta}, a, b)$, $X_i$ is the midpoint of the $i$-th fractile, $n$ is the number of fractiles used, and $\hat{\alpha}, \hat{\beta} > 0$. The parameters $a$ and $b$ are chosen by the user and $p_i$ is the probability associated with the $i$-th fractile [3].

Monte Carlo simulation is a method to model the uncertainty in a given system. Assume an uncertainty can be modeled by a known distribution; a computer then produces a large number of random numbers from that distribution. The utilities of the associated outcomes are then used to find the expected utility of the lottery [11].

Often multiple, correlated random variables must be modeled in the decision model. Sklar [44] states given a joint cumulative distribution function (CDF) $F(x_1, \ldots, x_n)$ with marginal CDFs $F_1(x_1), \ldots, F_n(x_n)$, the joint CDF can be written as a function of its marginals,

$$F(x_1, \ldots, x_n) = C[F_1(x_1), \ldots, F_n(x_n)]. \tag{2.4}$$

The function, $C$, is called a copula. If each marginal CDF, $F_i$, is continuous, then $C$ is a unique function for the given joint CDF [44].

Clemen and Reilly [10] describe how to create the joint multivariate random sample for a Monte Carlo simulation using the multivariate normal copula: 1) generate a vector of random numbers $(y_1, \ldots, y_n)$ from a multivariate normal random number generator using the desired correlation matrix, $\boldsymbol{R}$, 2) the standard normal distribution function, $\Phi(y_i)$ is calculated for each of the $n$ variables, 3) the inverse marginal distribution functions for each variable is then used to calculate the vector of required variates, $(F_1^{-1}[\Phi(y_1)], \ldots, F_n^{-1}[\Phi(y_n)])$ for the Monte Carlo simulation.

### 2.2.5 Multiattribute Utility Theory.

Two common methods for formulating multiattribute utility functions include creating a multiattribute value function using deterministic trade-offs between the attributes and then assessing a single attribute utility function over that value or by assessing conditional utility functions over each of the individual attributes and then combining these into a single multiattribute utility function by using various independence assumptions [1].

Some of the independence assumptions made while forming a multiattribute utility function include preferential independence, utility independence, and additive independence. To describe these independence assumptions, consider a multiattribute decision situation with up to three attributes, $y_1$, $y_2$, $y_3$ where each attribute falls within the range, $y_i \in [y_i^0, y_i^*]$, $i = 1, 2, 3$. Assume the attributes can be ordered such that $(y_1^0, y_2^0, y_3^0)$ is the least preferred prospect and $(y_1^*, y_2^*, y_3^*)$ is the most preferred prospect.

The attributes, $y_1$ and $y_2$, are said to be preferentially independent of $y_3$ if a given deterministic prospect $(y_1^{(2)}, y_2^{(2)}, y_3)$ is preferred to or indifferent to $(y_1^{(1)}, y_2^{(1)}, y_3)$ regardless of the value of $y_3$ [22]. Equation 2.5 displays this concept.

$$(y_1^{(2)}, y_2^{(2)}, y_3) \succeq (y_1^{(1)}, y_2^{(1)}, y_3), \forall y_3 \tag{2.5}$$

In a two attribute decision space with attributes, $y_1$ and $y_2$, $y_1$ is said to be utility independent of $y_2$ if the conditional preferences of $y_1$ given a certain value of $y_2$ do not depend on the value of $y_2$ [22]. In other words, the conditional utility function over $y_1$ given a fixed value of $y_2$ is a positive linear transformation of the conditional utility function over $y_1$ given $y_2$ is fixed at any other value [22]. If $y_1$ is utility independent of $y_2$, then the two-attribute utility function must be of the form in Equation 2.6,

$$u(y_1, y_2) = g(y_2) + h(y_2)u(y_1, y_2^{(1)}), \tag{2.6}$$

where $g(y_2)$ and $h(y_2)$ are both functions that depend on $y_2$ only and $u(y_1, y_2^{(1)})$ is the conditional utility function over $y_1$ given $y_2 = y_2^{(1)}$ [23].

Two attributes, $y_1$ and $y_2$, are said to be additive independent if the comparison between any two prospects depends only on the marginal preference structure of the two attributes. Consider two levels of each attribute, $y_1^{(1)}, y_1^{(2)}, y_2^{(1)}, y_2^{(2)}$. If $y_1$ and $y_2$

are additive independent, then the lottery with equal chances of the two prospects, $(y_1^{(1)}, y_2^{(1)})$ and $(y_1^{(2)}, y_2^{(2)})$ is equivalent to the lottery with the two prospects, $(y_1^{(1)}, y_2^{(2)})$ and $(y_1^{(2)}, y_2^{(1)})$, with equal chances [23].

If two attributes are additive independent, the two-attribute utility function is additive and can be written as in Equation 2.7.

$$u(y_1, y_2) = k_{y_1} u_{y_1}(y_1) + k_{y_2} u_{y_2}(y_2) \tag{2.7}$$

The utility function is normalized such that $u(y_1^0, y_2^0) = 0$ and $u(y_1^*, y_2^*) = 1$. The marginal utility function, $u_{y_i}(y_i)$, is normalized such that $u_{y_i}(y_i^0) = 0$ and $u_{y_i}(y_i^*) = 1$ $(i = 1, 2)$. The constants, $k_{y_1}$ and $k_{y_2}$ are calculated such that $k_{y_1} = u(y_1^*, y_2^0)$ and $k_{y_2} = u(y_1^0, y_2^*)$ [23].

Richard [39] defines multivariate risk aversion as the condition in which the decision maker prefers a lottery with an even chance of the prospects, $(y_1^{(1)}, y_2^{(2)})$ or $(y_1^{(2)}, y_2^{(1)})$, where $y_i^{(1)} < y_i^{(2)}$, $i = 1, 2$ to a lottery with even chances for $(y_1^{(1)}, y_2^{(1)})$ or $(y_1^{(2)}, y_2^{(2)})$. Indifference between these two lotteries is referred to as multivariate risk neutrality. A preference of the second lottery to the first is multivariate risk seeking behavior. Richard shows that the sign of the mixed partial derivative of the utility function indicates the multivariate risk preference expressed by the utility function [39]. Table 1 gives the results.

**Table 1. Sign of utility function's mixed partial derivative compared to multivariate risk preference**

| Sign of $\frac{\partial^2}{\partial y_1 \partial y_2} U(y_1, y_2)$ | Multivariate Risk Preference |
|---|---|
| - | averse |
| 0 | neutral |
| + | seeking |

### 2.2.6 Utility Transversality.

Matheson and Abbas introduce the concept of utility transversality in the multiattribute case where a utility function is assigned over a multiattribute value function [31]. Equation 2.8 shows the general formulation of such a utility function,

$$U(y_1, \ldots, y_n) = U_V(V(y_1, \ldots, y_n)), \tag{2.8}$$

where $y_1, \ldots, y_n$ are the attributes under consideration, $V(y_1, \ldots, y_n)$ is the deterministic value function over these attributes, and $U_V(V)$ is the utility function with respect to value [31].

Assuming that the utility and value functions are both monotonically increasing and twice differentiable, the risk aversion function with respect to a single attribute, $y_i, i = 1, \ldots, n$, is given in Equation 2.9,

$$\gamma_{y_i}^U = -\frac{U''_{y_i}}{U'_{y_i}}, \tag{2.9}$$

where $U''_{y_i} = \partial^2 U(y_1, \ldots, y_n)/\partial y_i^2$ and $U'_{y_i} = \partial U(y_1, \ldots, y_n)/\partial y_i$ [31].

Considering the two dimensional case where $U(y_1, y_2) = U_V(V(y_1, y_2))$ and following the chain rule of differentiation, Matheson and Abbas [31] show that the risk aversion with respect to a single variable is

$$\gamma_{y_1}^U = \gamma_V^U V'_{y_1} + \gamma_{y_1}^V. \tag{2.10}$$

The risk aversion with respect to value is $\gamma_V^U$. The partial derivative of value with respect to $y_1$ is $V'_{y_1} = \partial V(y_1, y_2)/\partial y_1$. Matheson and Abbas define $\gamma_{y_1}^V$ as the value function's contribution to the risk aversion with respect to $y_1$ as

$$\gamma_{y_1}^V = -\frac{V''_{y_1}}{V'_{y_1}}, \tag{2.11}$$

where $V''_{y_1} = \partial^2 V(y_1, y_2)/\partial y_1^2$ [31]. This is analogous to the definition of risk aversion shown in Equation 2.1.

The risk aversion with respect to a single attribute is defined completely by the risk aversion with respect to value and the form of the deterministic value function itself. Since the value function is deterministic and can be assessed by specifying the deterministic tradeoffs between attributes, then assessing the utility function over value determines the risk aversion function for all attributes [31].

Matheson and Abbas define the utility transversality relation as the relationship between the risk aversion functions of the various attributes and is shown in Equation 2.12 [31].

$$\gamma_{y_1}^U = (\gamma_{y_2}^U - \gamma_{y_2}^V)t + \gamma_{y_1}^V \tag{2.12}$$

The value, $t$, is the deterministic tradeoff function between the attributes $y_1$ and $y_2$ along an isopreference contour, calculated as in Equation 2.13 [31].

$$t(y_1, y_2) = \frac{V'_{y_1}}{V'_{y_2}} = -\frac{dy_2}{dy_1}\Big|_{\text{isopreference contour}} \tag{2.13}$$

### 2.2.7 Attribute Dominance.

Abbas and Howard introduced the concept of attribute dominant utility functions [4]. This special class of multiattribute utility functions satisfies four conditions.

Consider a two-attribute decision situation where a given prospect can be written as $(y_1, y_2)$ and the two attributes, $y_1$ and $y_2$, fall within the respective ranges, $y_1 \in [y_1^0, y_1^*]$ and $y_2 \in [y_2^0, y_2^*]$. Assume the attributes can be ordered such that $(y_1^0, y_2^0)$ is the least preferred prospect and $(y_1^*, y_2^*)$ is the most preferred. Assume the attributes are mutually preferentially independent and the prospects are arranged such that for any $y_1^{(1)}, y_1^{(2)}, y_2^{(1)}, y_2^{(2)}$, the following conditions hold [4].

$$
\begin{aligned}
&y_1^{(2)} \geq y_1^{(1)} \Rightarrow (y_1^{(2)}, y_2) \succeq (y_1^{(1)}, y_2) \forall y_2 \in [y_2^0, y_2^*] \text{ and} \\
&y_2^{(2)} \geq y_2^{(1)} \Rightarrow (y_1, y_2^{(2)}) \succeq (y_1, y_2^{(1)}) \forall y_1 \in [y_1^0, y_1^*]
\end{aligned}
\tag{2.14}
$$

Assume a multiattribute utility function $U_{y_1 y_2}(y_1, y_2)$ exists with range as given in Equation 2.15 [4].

$$
0 \leq U_{y_1 y_2}(y_1, y_2) \leq 1, \forall y_1 \in [y_1^0, y_1^*], y_2 \in [y_2^0, y_2^*] \tag{2.15}
$$

Based on this formulation, the utility of the prospects, $(y_1^0, y_2^0)$ and $(y_1^*, y_2^*)$, are as shown in Equation 2.16 [4].

$$
U_{y_1 y_2}(y_1^0, y_2^0) = 0,\ U_{y_1 y_2}(y_1^*, y_2^*) = 1 \tag{2.16}
$$

With the preceding assumptions, an attribute dominance utility function is one where a prospect, $(y_1, y_2)$, is a least preferred prospect if at least one of its attributes, $y_1$ or $y_2$, is at its minimum value. Equation 2.17 describes this condition [4].

$$U_{y_1 y_2}(y_1^0, y_2^0) = U_{y_1 y_2}(y_1^0, y_2) = U_{y_1 y_2}(y_1, y_2^0) = 0,$$
$$\forall y_1 \in [y_1^0, y_1^*], y_2 \in [y_2^0, y_2^*] \tag{2.17}$$

## 2.3 Response Surface Methodology

Response Surface Methodology (RSM) is a set of mathematical and statistical tools used to optimize systems [33]. RSM provides techniques to estimate a model of the system of interest by running an experiment varying the input variables of interest and measuring the response of interest [32]. A function is estimated from the results of such an experiment as shown in Equation 2.18

$$y = \boldsymbol{\beta}\mathbf{x} + \epsilon \tag{2.18}$$

where $y$ is the response in question, $\mathbf{x}$ is the vector of input variables, $\boldsymbol{\beta}$ is a vector of coefficients estimated from the experiment and $\epsilon$ is a random variable representing the random noise within the system [32].

Equation 2.18 can be extended to multiple responses using a vector of responses, $\mathbf{y}$, a matrix of coefficients, $\mathbf{B}$, and a vector of random noise variables, $\boldsymbol{\epsilon}$. Equation 2.19 shows such an equation.

$$\mathbf{y} = \mathbf{B}\mathbf{x} + \boldsymbol{\epsilon} \tag{2.19}$$

Classic RSM and regression techniques are built around systems occurring in nature or in man-made environments. Special considerations must be given when analyzing computer simulation models. Common random numbers are commonly used when comparing different simulation scenarios since they can sharpen the comparison between scenarios. When attempting to estimate the simulation model with a regression polynomial, non-overlapping pseudo-random number (PRN) streams must be used at the different design points so the outputs are independently identically distributed as RSM methods assume [27].

One other difference in a computer simulation experiment is that it can be accomplished sequentially without needing to consider blocking the the analysis. In a classic experiment, the experimenter would generally randomize the order of the design points to remove any ordering bias. If an augmented design were to be added to this experiment, the experimenter would have to analyze the data with that sequence in mind since the augmented design points were not randomized with the original set. A computer simulation experiment can be sequenced similarly without any additional concern to the change in analysis due to the augmented design. The order of the experiment does not affect the computer output if a non-overlapping random stream of PRNs is used to initiate any design points [27].

### 2.3.1 Desirability Function.

Harrington introduces the desirability function in order to find an optimal solution when optimizing multiple, often competing, objectives simultaneously [15]. The desirability function scales each objective measure, $y_i$, to a scale between 0 and 1, denoted by $d_i$. The overall desirability of a given solution is the geometric mean of the individual desirabilities as shown in Equation 2.20 [15]. The optimal solution is the vector of $y_i$'s that produces the largest overall desirability, $D$.

$$D = (d_1 d_2 \cdots d_n)^{\frac{1}{n}} \tag{2.20}$$

Derringer and Suich modify the function for the individual objective desirabilities so that they are more flexible [13]. Equation 2.21 shows the function for an individual objective's desirability where a maximum value of the response, $y_i$, is desired.

$$d_i = \begin{cases} 0, & y_i \leq L_i \\ \left[\frac{y_i - L_i}{T_i - L_i}\right]^{r_i}, & L_i < y_i < T_i \\ 1, & y_i \geq T_i \end{cases} \tag{2.21}$$

The parameters, $L_i$, $T_i$, and $r_i$, are provided by the decision maker. The parameter $L_i$ indicates the minimum acceptable value of $y_i$. Any value of $y_i$ below $L_i$ is unacceptable. This qualitative characteristic is expressed in the desirability function by forcing $d_i = 0$ which then forces the overall desirability to $D = 0$ as well. The parameter $T_i$ is an optimal value of $y_i$ or the point at which any more $y_i$ would give no additional value. The parameter $r_i$ controls how quickly the individual objective desirability increases from 0 to 1 as $y_i$ increases from $L_i$ to $T_i$. Figure 2 depicts how different values of $r_i$ affect the desirability function.

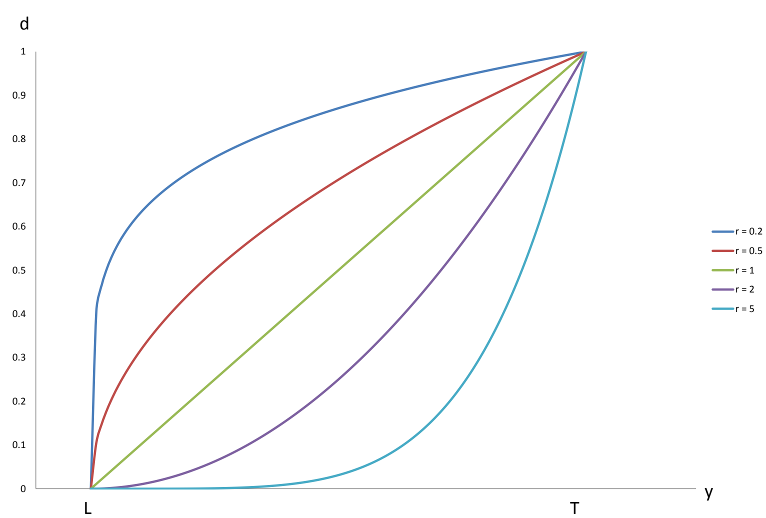


**Figure 2. Maximizing desirability functions, $d_i$, for various levels of $r_i$ [13]**

Equation 2.22 shows the function for an individual objective's desirability where a minimum value of the response, $y_i$, is desired.

$$d_i = \begin{cases} 1, & y_i \leq T_i \\ \left[\frac{y_i - U_i}{T_i - U_i}\right]^{r_i}, & T_i < y_i < U_i \\ 0, & y_i \geq U_i \end{cases} \tag{2.22}$$

The parameters, $U_i$, $T_i$, and $r_i$, are similarly provided by the decision maker. The parameter $U_i$ is the maximum value of $y_i$. Any value above this is unacceptable to the decision maker and will drive the overall desirability to $D = 0$. The parameter $r_i$ is similar to its use in the maximizing function. The parameter $T_i$ is the optimal value of $y_i$ or the point at which any level of $y_i$ below this would warrant little extra value [13].

At times, a specific target value is desired. In this case, a two-sided transformation is applicable as shown in Equation 2.23.

$$d_i = \begin{cases} 0, & y_i \leq L_i \\ \left[\frac{y_i - L_i}{T_i - L_i}\right]^{s_i}, & L_i < y_i \leq T_i \\ \left[\frac{y_i - U_i}{T_i - U_i}\right]^{t_i}, & T_i < y_i < U_i \\ 0, & y_i \leq U_i \end{cases} \tag{2.23}$$

The parameters, $L_i$ and $s_i$, are synonymous to $L_i$ and $r_i$ in equation 2.21. The parameters, $U_i$ and $t_i$, are synonymous to $U_i$ and $r_i$ in equation 2.22. The parameter $T_i$ is the optimal value of the objective, $y_i$. When $y_i = T_i$, the individual desiribility, $d_i = 1$ [13]. Figure 3 depicts two-sided transformations for various levels of $s_i$ and $t_i$.

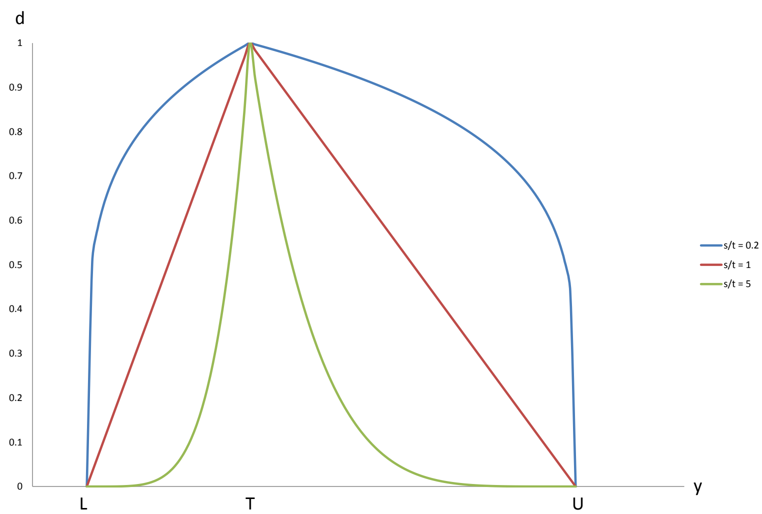


**Figure 3. Two-sided desirability functions, $d_i$, for various levels of $s_i$ and $t_i$ [13]**

### 2.3.2 Robust Optimization.

The optimal solution found in a multiple response situation using the desirability function does not take uncertainty into consideration and does not consider the variability of the responses [36].

Figure 4 illustrates the concept of a robust solution. The $x$-axis shows an input variable affecting a maximum response along the $y$-axis. At $x = x_2$, point B is clearly the global maximum within this range. However, as $x$ is allowed to vary from $x_2$ by $r$, the response drops $\Delta_B$, from $y_2$ to $y_2'$. Conversely, if the local optimum point A is considered and $x$ is allowed to vary from $x_1$ by $r$, the response drops $\Delta_A$, from $y_1$ to $y_1'$. In this situation, where $\Delta_A < \Delta_B$, point A can be considered more robust than point B [16].

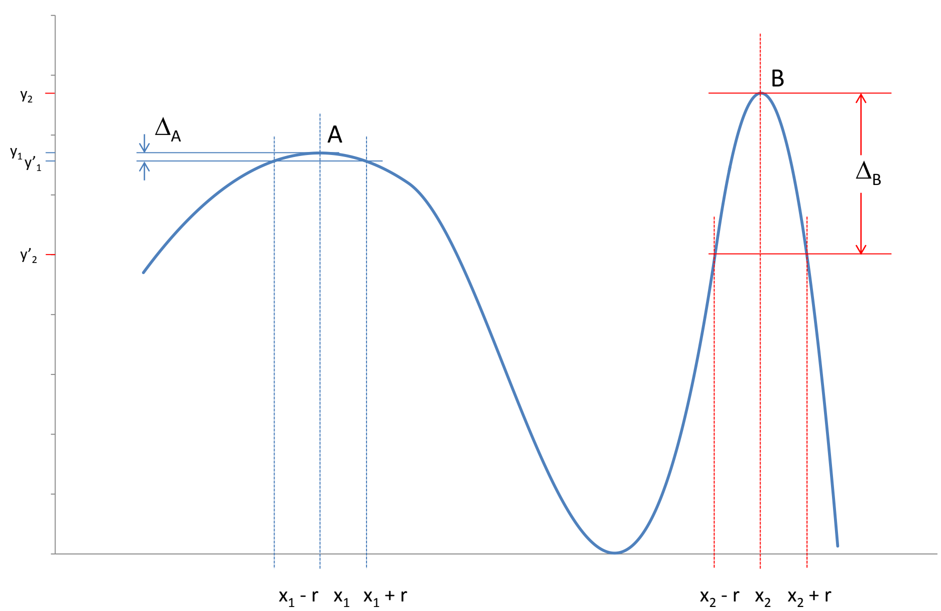


**Figure 4. Illustration of Robust Solution [16]**

From a decision analysis perspective, the preference between point A and point B is dependent upon the decision maker's risk preference. A sufficiently risk averse decision maker would prefer point A. A decision maker tending toward risk neutral or risk seeking behavior would prefer point B instead.

Taguchi introduces robust parameter design [46, 47] for situations where uncontrollable or nuisance factors exist and a solution that is insensitive to the variability of these nuisance factors is desired.

Vining and Myers present a constrained optimization method using predictive regression models for both the process mean and variance [51]. The robust solution is found by either constraining the mean while minimizing the variance or constraining the variance while optimizing the mean [51].

While these two methods focus on a single response of interest, Peterson, Miró-Quesado and del Castillo present a robust approach to a multiple response problem using Monte Carlo simulation [36]. After calculating the response equations to the seemingly unrelated regression (SUR) model, they calculate the probability a given set of inputs would give an acceptable set of output values using results from a Monte Carlo simulation.

## 2.4 Stability, Security, Transition, and Reconstruction Operations

The National Security Strategy (NSS) [34] outlines the overall security strategy for the nation. President Obama states the greatest threat to the American people is the possibility of violent extremists obtaining weapons of mass destruction. Engagement with other countries, especially emerging nations, is key to the NSS [34]. President Obama notes the need for an interagency approach to this end. Assisting developing countries join the world economy, manage their security needs, and assist their governments to lead their nations with an eye to human rights is imperative. The military's role includes excelling at counterterrorism and stability operations while maintaining readiness to address the full range of military operations [34].

Stability operations are essential to keeping a safe and secure environment for U.S. interests abroad. Department of Defense Instruction (DoDI) 3000.05 [48] states stability operations are a core U.S. military mission and should be conducted across the range of military operations. Further, U.S. military operations must strike "appropriate balance between offensive and defensive operations and stability operations in all phases" [49].

Military stability operations are planned based on effects based operations (EBO) as outlined by the United States Joint Forces Command [50]. A nation or region in question is characterized as a system of systems in terms of its political, military, economic, social, infrastructure, and information (PMESII) systems. The effects of diplomatic, informational, military and economic (DIME) instruments of national policy is then modeled to estimate the PMESII effects within the nation or region in question [50].

The current NSS is interested in developing diplomatic relations, security measures, and economic markets in African nations [34]. The U.S. is particularly concerned about assisting the Democratic Republic of Congo (DRC) in its efforts to create a secure, stable government. Concern still exists about human rights violations in the DRC after its recent presidential election [12].

## 2.5 National Operational Environment Model

The National Operational Environment Model (NOEM) is a simulation model representing a nation-state or region used to test a variety of courses of actions [42]. NOEM subscribes to the DIME and PMESII paradigm while modeling nation security and stability [43]. The main contributions to NOEM come from Richardson [40], Robbins [41], and Fensterer [14]. Richardson [40] applies system dynamics modeling techniques to a Stability and Reconstruction effort at a national level. This allows a statistical analysis of various high-level policy choices.

Robbins [41] improves upon this idea by developing a more detailed model, the Stabilization and Reconstruction Operations Model (SROM), which allows for more comprehensive analysis of the nation-building efforts. SROM models DIME and PMESII interactions at the sub-national, regional level which allows testing of a wide variety of policy options on a national or regional basis.

Fensterer [14] uses a Value Focused Thinking strategy to capture the important values of nation state stability. Using Department of Defense (DoD) guidance, Fensterer suggests five fundamental values of stability: Economy, Governance, Rule of Law, Security, and Social Well-Being.

Based on this research, SROM was re-engineered at the Air Force Research Laboratory to become NOEM [42]. NOEM contains several modules to model the complex and critical DIME and PMESII interactions within a region. Although most modules contain deterministic models, an Agent-Based model represents the population and provides a stochastic element to the overall model [42].

# III. Methodology

## 3.1 Research Methodology

This chapter develops the methodology used to examine the desirability function and the methodology used to find a robust optimum solution from a decision analysis perspective. Section 3.2 an analysis of the desirability function from a decision analysis perspective. Section 3.2.1 analyzes the desirability function as a value function. Sections 3.2.2 and 3.2.3 analyze a utility function assessed over the desirability function. Section 3.3 develops the methodology taken to find a robust decision point within a multiple response system.

## 3.2 Desirability Function Analysis

This section presents an analysis of Derringer and Suich's desirability function [13] from a decision analysis perspective. Two different cases are examined. In the first case, assume the desirability function is a value function. In the second case, assume the desirability function is a utility function. The analysis focuses on the two-dimensional case with responses, $y_1$ and $y_2$. The analysis is generalized to an $n$-dimensional case, which is presented in Appendix A.

The desirability function for the 2-dimensional case is

$$D = (d_1 d_2)^{\frac{1}{2}}. \tag{3.1}$$

Consider a system with $n$ inputs, $(x_1, \ldots, x_n)$. Each input, $x_i$ falls within the experimental region defined by $x_i \in [x_i^0, x_i^*]$, $i = 1, \ldots, n$. These inputs cause changes in two responses, $y_1$ and $y_2$ which each fall within the range, $y_i \in [y_i^0, y_i^*]$, $i = 1, 2$. This analysis focuses on the case where a maximum response is desired as displayed in Equation 2.21 with no loss of generality since the other forms of the desirability function can be transformed into this case. Only the non-trivial piece of Equation 2.21 where $L_i < y_i < T_i, i = 1, 2$ is considered. Equation 3.2 displays this non-trivial piece.

$$d_i = \left[\frac{y_i - L_i}{T_i - L_i}\right]^{r_i},\ L_i < y_i < T_i \tag{3.2}$$

Where $[L_i, T_i] \subseteq [y_i^0, y_i^*]$, $i = 1, 2$. This forces $0 < D < 1$.

### 3.2.1 Desirability function as value function.

Consider the case in which the desirability function is used as a value function. The desirability function can be used as a value function if the decision maker assumes the expected responses from the regression equations are deterministic. The deterministic trade-off between the two attributes can then be examined using the tradeoff function as defined in Chapter II.

$$t(y_1, y_2) = \frac{D'_{y_1}}{D'_{y_2}} = -\frac{dy}{dx}|_{\text{isopreference contour}} \tag{3.3}$$

The deterministic tradeoffs between the two attributes can then be stated.

$$t(y_1, y_2) = \frac{r_1}{r_2}\left[\frac{y_2 - L_2}{y_1 - L_1}\right] \tag{3.4}$$

Equation 3.4 indicates how many units of $y_2$ the decision maker is willing to give up in order to increase $y_1$ by one unit, at the point $(y_1, y_2)$. In this case, the tradeoff is a ratio of the two exponents $r_1$ and $r_2$ multiplied by the ratio of the differences between the two current values of $y_i$ and their respective minimum acceptable values, $L_i$. Table 2 shows how the tradeoff changes as each response or parameter is increased while all others are held constant. Interestingly, the two target values, $T_1$ and $T_2$, cancel out of the function when the ratio of the two partial derivatives of the overall desirability function is calculated.

**Table 2. Tradeoff varies with respect to different variables increasing**

| **Variable increasing (all else constant)** | **Effect on $t(y_1, y_2)$** |
|---|---|
| $y_1$ | decreases |
| $y_2$ | increases |
| $r_1$ | increases |
| $r_2$ | decreases |
| $L_1$ | increases |
| $L_2$ | decreases |

The desirability function tends to move the responses away from their minimum values. When a response is pushed near its minimum value, its individual desirability, $d_i$, tends to zero.

For a constant value of $y_2$, $t(y_1, y_2)$ decreases as $y_1$ increases. As $y_1$ increases from its minimum, $d_1$ increases from zero which increases the overall desirability. The further $y_1$ is from its minimum, the less $y_2$ a decision maker is willing to give up for further increases in $y_1$.

For a constant value of $y_1$, $t(y_1, y_2)$ increases as $y_2$ increases. The same argument applies as above. The further $y_2$ is from its minimum, the more $y_2$ a decision maker is willing to give up for further increases in $y_1$.

Consider the following example where $y_2$ is 10 units above its minimum value, $y_1$ is two units over its minimum, and $r_1 = r_2 = 1$. The decision maker is willing to reduce $y_2$ by five units to increase $y_1$ by one unit. See Equation 3.5.

$$t(L_1 + 2, L_2 + 10) = \frac{1}{1}\left[\frac{10}{2}\right] = 5 \tag{3.5}$$

Now let $y_2$ be one unit above its minimum value not 10 units, as above and keep all other values the same. The decision maker is only willing to reduce $y_2$ by 0.5 units to increase $y_1$ by one unit. Equation 3.6 depicts this result. After $y_2$ is reduces from 10 to 1, the decision maker does not have as much excess $y_2$ to pay in order to increase $y_1$. The decreased level of $y_2$ results in a lower tradeoff between the two attributes.

$$t(L_1 + 2, L_2 + 1) = \frac{1}{1}\left[\frac{1}{2}\right] = 0.5 \tag{3.6}$$

Increasing $r_1$ increases the trade-off. As $r_1$ increases, $d_1$ decreases for a given value of $y_1$. A higher $r_1$ implies the higher values of $y_1$ are worth a premium; the decision maker is willing to decrease more of $y_2$ in order to increase $y_1$. A smaller $r_1$ allows a much lower $y_1$ to achieve higher desirability.

Increasing $r_2$ decreases the tradeoff. As $r_2$ increases, $d_2$ decreases for a given value of $y_2$. A higher $r_2$ implies the higher values of $y_2$ are worth a premium; the decision maker is willing to decrease $y_2$ less in order to increase $y_1$.

Increasing $L_1$ increases the tradeoff. The minimum level of $y_1$ is being increased, forcing $d_1$ closer to zero for a fixed $y_1$. As $y_1$ approaches its minimum value (by raising that minimum aspiration level), the decision maker is willing to decrease $y_2$ more to increase $y_1$.

Increasing $L_2$ decreases the tradeoff. The minimum level of $y_2$ is being increased, forcing $d_2$ closer to zero for a fixed $y_2$. As $y_2$ approaches its minimum value (by raising that minimum aspiration level), the decision maker is willing to decrease $y_2$ less to increase $y_1$.

This tradeoff analysis conflicts with the analysis Kros and Mastrangelo [29] apply to the desirability function. Their analysis incorrectly takes the partial derivative of the desirability function with respect to $d_i$ and concludes that the desirability function implies a constant trade-off of one [29]. Their analysis considers the trade-off with respect to the two competing desirabilities. However, the trade-off between two attributes needs to be calculated with respect to the attributes themselves [31], not their desirability. In this case, that is with respect to the responses, $y_1$ and $y_2$.

### 3.2.2 Assessing a utility function over the desirability function.

When uncertainty is present in a decision situation, a utility function is required for proper assessment. As described in Chapter II, one method of formulating a multiattribute utility function is to assign a single attribute utility function over a multiattribute value function. Assume a 2-dimensional deterministic desirability function (Equation 3.1). Consider the case of constant risk aversion over desirability as a value function. Assume the following utility function

$$U = 1 - e^{-\gamma D}, \tag{3.7}$$

where $D$ is the value of the desirability function and $\gamma$ is the constant risk aversion with respect to this desirability. Assume the decision maker is risk averse with respect to desirability (i.e., $\gamma > 0$). Howard [19] notes through practical experience that most decision makers are risk averse in their attitude for risk taking.

Following the concepts developed by Matheson and Abbas [31], the risk aversion function for each attribute is expressed. The utility function's risk aversion with respect to the first response, $y_1$, is examined first. The risk aversion with respect to the second response follows.

The risk aversion of the utility function with respect to the first attribute, $y_1$, is

$$\gamma_{y_1}^U = \gamma_D^U D'_{y_1} + \gamma_{y_1}^D. \tag{3.8}$$

Consider each term in Equation 3.8. The risk aversion for the utility function with respect to the desirability function is simply

$$\gamma_D^U = \gamma. \tag{3.9}$$

The partial derivative of $D$ with respect to $y_1$ is

$$D'_{y_1} = \frac{r_1}{2(T_1 - L_1)} \left[\frac{y_1 - L_1}{T_1 - L_1}\right]^{\frac{r_1}{2}-1} \left[\frac{y_2 - L_2}{T_2 - L_2}\right]^{\frac{r_2}{2}}. \tag{3.10}$$

The contribution of the desirability function to the risk aversion of the attribute $y_1$ is

$$\gamma_{y_1}^D = \frac{1 - \frac{r_1}{2}}{y_1 - L_1}. \tag{3.11}$$

Substituting these expressions into Equation 3.8 provides the risk aversion expressed by this utility function with respect to $y_1$.

$$\gamma_{y_1}^U = \frac{\gamma r_1}{2(y_1 - L_1)} \left[\frac{y_1 - L_1}{T_1 - L_1}\right]^{\frac{r_1}{2}} \left[\frac{y_2 - L_2}{T_2 - L_2}\right]^{\frac{r_2}{2}} + \frac{1 - \frac{r_1}{2}}{y_1 - L_1} \tag{3.12}$$

From Equation 3.12, it is clear that the risk aversion with respect to $y_1$ is dependent on the ranges of both responses, the values of both responses, the exponent assigned to each response, and the constant risk aversion coefficient in the utility function over desirability. Based on the assumptions made in this case, every segment of this expression is positive with the exception of the numerator of the desirability function's contribution to risk aversion, $1 - \frac{r_1}{2}$. The sign of this contribution depends on the exponent $r_1$ and the sign switches at $r_1 = 2$. Examining the cases of $r_1$ less than, equal to, and greater than two provides further insight.

For $r_1 < 2$, each component of Equation 3.12 is positive; therefore, the utility function indicates risk aversion with respect to $y_1$. That being said, this aversion to risk varies with respect to changes in the inputs to the risk aversion function. Table 3 shows how risk aversion changes with respect to individual variables increasing, assuming all other inputs remain constant.

**Table 3. Risk aversion varies with respect to different variables increasing when $r_1 < 2$**

| **Variable increasing (all else constant)** | **Effect on $\gamma^U_{y_1}$** |
|---|---|
| $\gamma$ | increases |
| $y_1$ | decreases |
| $y_2$ | increases |
| $r_1$ | varies |
| $r_2$ | decreases |
| $L_1$ | increases |
| $L_2$ | decreases |
| $T_1$ | decreases |
| $T_2$ | decreases |

As $\gamma$, the risk aversion coefficient with respect to desirability, increases, showing increased risk aversion over desirability, the risk aversion with respect to an individual attribute increases.

As $y_1$ increases, the utility function displays a decreased risk aversion to $y_1$. The closer $y_1$ is to its target value, the decision maker can better afford taking a risk in $y_1$.

The variables associated with the other response, $y_2$, $r_2$ and $L_2$, affect risk preference due to the interaction within the desirability function. As mentioned in Chapter II and explicitly indicated by $\gamma^U_{y_1}$, the risk aversion with respect to a single attribute is defined in part by the form of the value function. In this case, the desirability function is a value function. The risk attitude reaction to increases in $y_2$ and $r_2$ match exactly with the tradeoff reactions in Table 2.

How $\gamma_{y_1}^U$ varies with respect to $r_1$ is more complicated. Equation 3.13 shows the partial derivative of the risk aversion, $\gamma_{y_1}^U$, with respect to $r_1$.

$$\frac{\partial}{\partial r_1}\gamma_{y_1}^U = \frac{1}{4(y_1 - L_1)}\left(\gamma\left[\frac{y_1 - L_1}{T_1 - L_1}\right]^{\frac{r_1}{2}}\left[\frac{y_2 - L_2}{T_2 - L_2}\right]^{\frac{r_2}{2}}\left[2 + r_1 \ln\left[\frac{y_1 - L_1}{T_1 - L_1}\right]\right] - 2\right) \tag{3.13}$$

This partial derivative is not strictly positive or negative. The sign of this partial derivative depends on the expression within the parentheses. If $d_1 \leq e^{-2}$, then $\frac{\partial}{\partial r_1}\gamma_{y_1}^U < 0$. However, the converse is not true.

Figure 5 shows isopreference curves for various values of $\gamma$, with $d_1$ on the horizontal axis and $d_2$ on the vertical axis. If $(d_1, d_2)$ lies on an isopreference curve, $\frac{\partial}{\partial r_1}\gamma_{y_1}^U = 0$. If $(d_1, d_2)$ lies southwest of this isopreference curve, $\frac{\partial}{\partial r_1}\gamma_{y_1}^U < 0$ and if $(d_1, d_2)$ lies northeast of this curve, $\frac{\partial}{\partial r_1}\gamma_{y_1}^U > 0$.

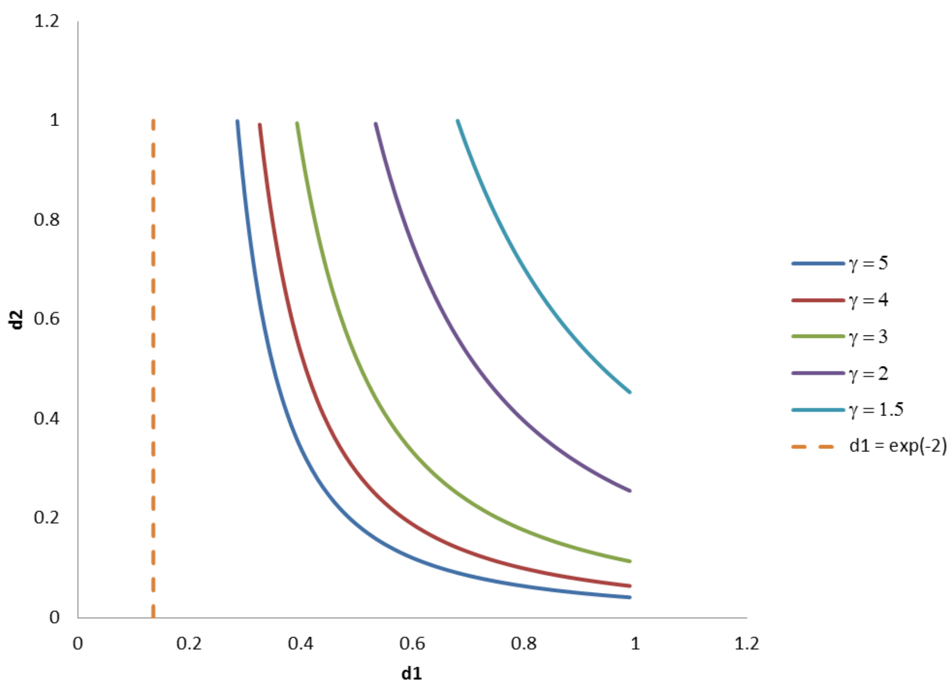


**Figure 5. Isopreference curves of $\gamma$ with respect to $(d_1, d_2)$ that make $\frac{\partial}{\partial r_1}\gamma_{y_1}^U = 0$**

The decision maker must be aware of unintentional consequences of adjusting $r_1$. Figure 5 shows how increasing $r_1$ increases risk aversion with respect to $y_1$ for a particular area of the desirability while simultaneously decreasing risk aversion with respect to $y_1$ for its complimentary area. As $\gamma$ decreases, the decision maker is moving towards risk neutrality. This action also decreases the area where $\frac{\partial}{\partial r_1}\gamma_{y_1}^U > 0$ and increases the area where $\frac{\partial}{\partial r_1}\gamma_{y_1}^U < 0$

As $L_1$ increases, risk aversion increases. The minimum level of $y_1$ is being increased, forcing $d_1$ closer to zero for a fixed $y_1$. As $y_1$ approaches its minimum value (by raising that minimum aspiration level) the utility function expresses more risk aversion with respect to $y_1$.

As $T_1$ increases, risk aversion decreases. Since a fixed value of $y_1$ is being placed further from its target, the utility function expresses less risk aversion. Increasing the size of the interval $[L_1, T_1]$ decreases risk aversion with respect to $y_1$ while decreasing the size of the interval $[L_1, T_1]$ increases risk aversion with respect to $y_1$.

As $T_2$ increases, risk aversion decreases. Since a fixed value of $y_2$ is being placed further from its target, the utility function expresses less risk aversion.

For $r_1 = 2$, parts of Equation 3.12 cancel out, leaving only positive expressions. It follows that the utility function expresses risk aversion with respect to $y_1$. This aversion to risk varies with respect to changes in the inputs to the risk aversion function. Table 4 shows how risk aversion changes with respect to individual variables increasing, assuming all other inputs remain constant.

The only difference between Table 4 and Table 3 (denoted by the asterisk) is that the response $y_1$ has no effect on the risk aversion expressed with respect to $y_1$. This is due to the fact that the exponent $r_1/2 = 1$ and $y_1$ then cancel out of $\gamma_{y_1}^U$. When $r_1 = 2$, the desirability function is said to be risk-contribution neutral with respect to $y_1$ since $\gamma_{y_1}^D = 0$ [31].

**Table 4. Risk aversion varies with respect to different variables increasing when $r_1 = 2$**

| **Variable increasing (all else constant)** | **Effect on $\gamma^U_{y_1}$** |
|---|---|
| $\gamma$ | increases |
| $y_1$ | no effect* |
| $y_2$ | increases |
| $r_2$ | decreases |
| $L_1$ | increases |
| $L_2$ | decreases |
| $T_1$ | decreases |
| $T_2$ | decreases |

For $r_1 > 2$, the desirability function's contribution to risk aversion, $\gamma^D_{y_1}$, is negative. The utility function can express risk seeking, risk neutral, or risk aversion with respect to $y_1$. Table 5 shows how risk aversion changes with respect to individual variables increasing, assuming all other inputs remain constant.

**Table 5. Risk aversion varies with respect to different variables increasing when $r_1 > 2$**

| **Variable increasing (all else constant)** | **Effect on $\gamma^U_{y_1}$** |
|---|---|
| $\gamma$ | increases |
| $y_1$ | increases* |
| $y_2$ | increases |
| $r_1$ | varies |
| $r_2$ | decreases |
| $L_1$ | varies* |
| $L_2$ | decreases |
| $T_1$ | decreases |
| $T_2$ | decreases |

When comparing the results shown in Table 5 with the results shown in Table 3, it is seen that the utility function displays increasing risk aversion with respect to $y_1$ as the response, $y_1$, increases. Increasing risk aversion is not how most decision makers prefer their risk aversion to be modeled.

Another change in this case is how $L_1$ affects risk aversion with respect to $y_1$. Equations 3.14 and 3.15 display the partial derivative of $\gamma^U_{y_1}$ with respect to $L_1$.

$$\frac{\partial}{\partial L_1}\gamma^U_{y_1} = \frac{1}{2(y_1-L_1)^2}\left[\gamma r_1 \left[\frac{y_1-L_1}{T_1-L_1}\right]^{\frac{r_1}{2}} \left[\frac{y_2-L_2}{T_2-L_2}\right]^{\frac{r_2}{2}} \left(1-\frac{r_1(T_1-y_1)}{2(T_1-L_1)}\right)+2-r_1\right] \tag{3.14}$$

$$= \frac{1}{2(y_1-L_1)^2}\left[\gamma r_1 D\left(1-\frac{r_1(T_1-y_1)}{2(T_1-L_1)}\right)+2-r_1\right] \tag{3.15}$$

This partial derivative is no longer monotonic. Table 6 displays the sign of $\frac{\partial}{\partial L_1}\gamma^U_{y_1}$ based on the value of $r_1$. When $r_1 \in [2, \frac{2(T_1-L_1)}{T_1-y_1}]$ the sign of $\frac{\partial}{\partial L_1}\gamma^U_{y_1}$ can be either positive or negative depending on the relationship of the parameters within the brackets of Equation 3.15.

**Table 6. Sign of $\frac{\partial}{\partial L_1}\gamma^U_{y_1}$ based on $r_1$**

| Value of $r_1$ | Sign of $\frac{\partial}{\partial L_1}\gamma^U_{y_1}$ |
|---|---|
| $r_1 < 2$ | + |
| $2 < r_1 < \frac{2(T_1-L_1)}{T_1-y_1}$ | +/0/- |
| $r_1 > \frac{2(T_1-L_1)}{T_1-y_1}$ | - |

The decision maker must be aware of the unintended consequences adjusting the bounds on responses has on risk preference.

### 3.2.3 Conjugate Desirability Function.

The exponential utility function over desirability example produced a case where the utility function over value was from a different family of utility functions than the marginal utility functions over the attributes. In many cases, a decision maker may want to express the same risk preference for all the attributes as is expressed for the given value function. In considering this case, Matheson and Abbas present the idea of a conjugate value function [31].

A value function is said to be conjugate to a utility function over value if the marginal utility function over an attribute is of the same family as the utility function over value [31].

Consider a logarithmic utility function over desirability as shown in Equation 3.16.

$$U = \ln D \tag{3.16}$$

The risk aversion with respect to desirability is

$$\gamma_D^U = \frac{1}{D}. \tag{3.17}$$

The partial derivative of $D$ with respect to $y_1$, $D'_{y_1}$, and the contribution of the desirability function to the risk aversion of $y_1$, $\gamma_{y_1}^D$ are shown in Equations 3.10 and 3.11 respectively. The risk aversion with respect to $y_1$ is

$$\gamma_{y_1}^U = \frac{1}{y_1 - L_1}. \tag{3.18}$$

Matheson and Abbas state "the product value function is conjugate to the logarithmic utility function" [31]. It follows that the desirability function is conjugate to the logarithmic utility function.

### 3.2.4 Desirability function as a utility function.

If the desirability function itself is assumed to be a utility function, it follows Abbas and Howard's definition of an attribute dominance utility function [4] since its value is zero if any of the attributes is at or below its minimum acceptable value. When considering the desirability function in this manner, it is denoted as $D^d$. Equation 3.19 shows the two-dimensional case when $L_i < y_i < T_i, i = 1, 2$.

$$D^d_{y_1 y_2}(y_1, y_2) = \left[\frac{y_1 - L_1}{T_1 - L_1}\right]^{\frac{r_1}{2}} \left[\frac{y_2 - L_2}{T_2 - L_2}\right]^{\frac{r_2}{2}} \tag{3.19}$$

Equation 3.20 gives the marginal utility function for the first attribute.

$$D^d_{y_1} = D^d_{y_1 y_2}(y_1, T_2) = \left[\frac{y_1 - L_1}{T_1 - L_1}\right]^{\frac{r_1}{2}} \tag{3.20}$$

Equation 3.21 gives the conditional utility function for the second attribute given a certain level of the first attribute.

$$D^d_{y_2|y_1}(y_2|y_1) = \frac{D^d_{y_1 y_2}(y_1, y_2)}{D^d_{y_1}} = \left[\frac{y_2 - L_2}{T_2 - L_2}\right]^{\frac{r_2}{2}} \tag{3.21}$$

Since Equation 3.21 does not depend on the value of $y_1$, the conditional utility function is equal to the marginal utility function and it is seen that the desirability function displays utility independence of the two attributes. The desirability function can then be expressed as in Equation 3.22.

$$D^d_{y_1 y_2}(y_1, y_2) = D^d_{y_1}(y_1) D^d_{y_2}(y_2) \tag{3.22}$$

With this assumption of utility independence, the analyst can simply assess the marginal utility functions of each attribute separately and then multiply them to create the overall desirability function. Equation 3.22 is a special case of Equation 2.6 which depicts the general case of a utility function expressing utility independence.

The attribute $y_1$ is utility independent of $y_2$ if the conditional preferences of $y_1$ given a certain value of $y_2$ do not depend on the value of $y_2$ [22]. If the attributes are not utility independent, the analyst should not use the desirability function as a utility function.

Equation 3.23 gives the risk aversion of the marginal utility function of the first attribute.

$$\gamma_{y_1} = \frac{1 - \frac{r_1}{2}}{y_1 - L_1} \tag{3.23}$$

Kros and Mastrangelo present erroneous conclusions regarding the risk aversion of the desirability function [29]. For $r_i = 2$, the desirability function reflects risk neutrality with respect to $y_i$. For $r_i < 2$ the desirability function reflects decreasingly risk averse behavior. For $r_i > 2$, the desirability function reflects decreasingly risk seeking behavior. This breakpoint in modeling risk preference is dependent on the number of objectives. When generalized to $n$-dimensions, the desirability function reflects risk neutrality when $r_i = n$. The desirability function reflects risk averse and risk seeking behavior when $r_i < n$ and $r_i > n$ respectively.

The mixed partial derivative of the desirability function is shown in Equation 3.24.

$$\frac{\partial^2 D^d_{y_1 y_2}(y_1, y_2)}{\partial y_1 \partial y_2} = \frac{r_1 r_2}{4(y_1 - L_1)(y_2 - L_2)} \left[\frac{y_1 - L_1}{T_1 - L_1}\right]^{\frac{r_1}{2}} \left[\frac{y_2 - L_2}{T_2 - L_2}\right]^{\frac{r_2}{2}} \tag{3.24}$$

This mixed partial derivative is always positive over the range $L_i < y_i < T_i, i = 1, 2$. Therefore, the desirability function displays multivariate risk seeking behavior [39]. This type of behavior makes sense in an attribute dominance utility function, since the overall utility is zero if any one of the attributes is at its minimum level.

The fact that the desirability function sets the overall desirability to zero if any of its attributes is at or below its minimum values, regardless of the values of the remaining attributes merits discussion. Consider the following three attribute relation.

$$D(L_1 + \epsilon, L_2 + \epsilon, L_3 + \epsilon) > D(L_1 - \epsilon, T_2, T_3) = 0, \tag{3.25}$$

where $\epsilon > 0$ is small. Consider the right hand side of Equation 3.25. Even with the second two attributes at their target values, since the first attribute is slightly below its minimum value, its desirability is zero. Consider the left hand side of Equation 3.25. With all three attributes slightly above their minimum values, the desirability is a small positive number and is therefore greater than the right hand side. Care must be taken to ensure a valid minimum value is decided upon before applying the desirability function. An elevated minimum value could reject a potentially acceptable decision point. Moreover, changing the minimum values of the responses also affects the trade-off and risk aversion functions. In decision analysis, a value (or utility) function is stipulated over any valid domain of interest in the attempt to avoid such errors (i.e., $L_i = y_i^0$ and $T_i = y_i^*$, $\forall i$).

## 3.3 A Decision Analysis Perspective on Robust Optimization

A set of regression equations is developed from an experimental design over the decision space. Once the decision maker selects the proper user-defined parameters, the maximum desirability is calculated and its location found within the feasible region.

Since the desirability function is not differentiable throughout its range, a non-gradient optimization algorithm must be used. The Hooke and Jeeves Direct Search algorithm [17, 20, 8] is implemented in Excel using Visual Basic for Applications (VBA). Since the starting point is critical when using such an algorithm over such un-smooth surfaces, multiple starting points are chosen throughout the feasible region. This is accomplished by taking a random sampling of the feasible region or by dividing the feasible region into a grid of equally spaced vertices along each dimension of the decision space. The code iterates through each starting point for the direct search algorithm. The code outputs the location and desirability of all local maxima. To confirm the results of the direct search algorithm, the local maxima is also calculated using the differential evolution algorithm within Matlab [38]. Once the deterministic local maxima are calculated, the Direct Search algorithm is re-run calculating the desirability function in a Monte Carlo simulation.

Clemen and Reilly discuss using regression equations for decision analysis. These equations estimate the conditional expected value of a response given a set of input variables. The difference between the observed value and the predicted value of the response is the residual. This set of residuals can be used to estimate the continuous distribution function (CDF) of the random error term, $\epsilon$, as described in Equation 2.18 in Chapter II [11]. This CDF is approximated with a Beta distribution using the residuals as the fractile midpoints in Matlab's *fminsearch* function as described in Abbas et al [3] and reviewed in Section 2.2.4.

The correlation of the residuals, $\boldsymbol{R}$, is calculated and then the random samples for the Monte Carlo simulation is calculated using the multivariate normal copula [10].

Matlab's *mvnrnd* function generates a matrix of random vectors from a multivariate standard normal distribution with correlation $\boldsymbol{R}$. Equation 3.26 displays how the vector of random samples $(\epsilon_1, \ldots, \epsilon_n)$ is calculated,

$$(\epsilon_1, \ldots, \epsilon_n) = (F_1^{-1}[\Phi(y_1)], \ldots, F_n^{-1}[\Phi(y_n)]), \tag{3.26}$$

where $\Phi$ is the standard normal CDF and $F_i^{-1}$, $i = 1, \ldots, n$ is the $i$-th inverse Beta marginal CDF estimated from the residuals.

Using each local maximum as the starting point, the solution to

$$\max_{\boldsymbol{y^{(1)}}} \sum_{i=1}^{n} \left( \frac{1}{n} D(\boldsymbol{y^{(1)}} + \boldsymbol{\epsilon_i}) \right) \tag{3.27}$$

is calculated where $n$ is the number of random samples used in the Monte Carlo simulation, $\boldsymbol{\epsilon_i}$ is $i$-th vector random samples and $\boldsymbol{y^{(1)}}$ is the vector of responses found with the highest expected desirability. The global maximum is the robust solution point for the system. Sensitivity analysis indicates the robustness of the solution.

# IV. Wire Bonding Experiment Findings

## 4.1 Multi-response optimization of semiconductor manufacturing process

Del Castillo et al. [9] introduce an experiment to illustrate a multi-response optimization problem using the desirability function. The experiment is executed to optimize a wire bonding process in the semiconductor industry. The experiment has three input factors and six responses. Table 7 contains the three factors and their low and high values used in the experiment. Table 8 contains the response descriptions and their acceptable ranges and target values.

**Table 7. Factors and levels for example experiment [9]**

| Factor | Factor Name | Levels Low | Levels High |
|---|---|---|---|
| A | Flow Rate | 40 | 120 |
| B | Flow Temp | 200 | 450 |
| C | Block Temp | 150 | 350 |

**Table 8. Response descriptions and ranges for multiple response example [9]**

| Response - description | Min | Max | Target |
|---|---|---|---|
| $y_1$ = maximum temperature at Position A | 185 | 195 | 190 |
| $y_2$ = beginning bond temperature at position A | 170 | 195 | 185 |
| $y_3$ = finish bond temperature at position A | 170 | 195 | 185 |
| $y_4$ = maximum temperature at position B | 185 | 195 | 190 |
| $y_5$ = beginning bond temperature at position B | 170 | 195 | 185 |
| $y_6$ = finish bond temperature at position B | 170 | 195 | 185 |

Excel Analysis ToolPak is used to run an ordinary least squares (OLS) multiple linear regression on the data del Castillo et al. provide from the experiment to estimate the response equations [9].

$$\hat{y}_1 = -18.404 + 0.567x_2 + 0.530x_3 - 0.002x_2x_3 \quad (4.1)$$

$$\hat{y}_2 = 37.463 + 0.150x_1 + 0.173x_2 + 0.141x_3 \quad (4.2)$$

$$\hat{y}_3 = 33.413 + 0.166x_1 + 0.128x_2 + 0.204x_3 \quad (4.3)$$

$$\hat{y}_4 = 45.616 + 0.767x_1 + 0.065x_2 + 0.079x_3 - 0.008x_1^2 + 0.002x_1x_2 \quad (4.4)$$

$$\hat{y}_5 = 51.443 + 0.230x_1 + 0.046x_2 + 0.165x_3 - 0.004x_1^2 + 0.001x_1x_2 \quad (4.5)$$

$$\hat{y}_6 = 36.066 + 0.239x_1 + 0.049x_2 + 0.274x_3 - 0.003x_1^2 + 0.001x_1x_2 \quad (4.6)$$

## 4.2 Deterministic optimum setting

The two-sided desirability function from Equation 2.23 is used to find the optimum operating environment within the experimental region described in Table 7. Using the Min, Max, and Target values from Table 8 as the values of $L_i$, $U_i$, $T_i$, $i = 1, \ldots, 6$ and Equations 4.1 through 4.6, the Hooke-Jeeves (HJ) algorithm finds the maximum desirability and its operating solution for various risk preferences. Table 9 contains the exponent parameters used to express various risk attitudes.

**Table 9. Exponent parameters used to express various risk attitudes**

| | Risk Attitude | | |
|---|---|---|---|
| | **Averse** | **Neutral** | **Seeking** |
| $s_i, t_i$ | 3 | 6 | 9 |

In each of the risk attitude cases described in Table 9, the same two local optimum solutions are found. Adjusting the exponents $s_i$ and $t_i$ ($i = 1, \ldots, 6$) such that $s_i = t_i$ ($i = 1, \ldots, 6$) will not change the optimum solution because the trade-off functions, $t(y_i, y_j)$, $i \neq j$ does not change.

Table 10 displays the two local optimum solutions and their predicted responses. The desirability values shown are from the risk neutral case where $s_i = t_i = 6$ $(i = 1, \ldots, 6)$. The first column contains the global optimum solution.

**Table 10. Local deterministic operating solutions**

| | Local Optimum Points | |
|---|---|---|
| $\boldsymbol{x_1}$ | 120.000 | 82.192 |
| $\boldsymbol{x_2}$ | 436.168 | 450.000 |
| $\boldsymbol{x_3}$ | 320.297 | 330.453 |
| $\boldsymbol{\hat{y}_1}$ | 186.391 | 185.939 |
| $\boldsymbol{\hat{y}_2}$ | 176.162 | 174.318 |
| $\boldsymbol{\hat{y}_3}$ | 174.378 | 171.955 |
| $\boldsymbol{\hat{y}_4}$ | 193.277 | 192.861 |
| $\boldsymbol{\hat{y}_5}$ | 173.139 | 172.910 |
| $\boldsymbol{\hat{y}_6}$ | 184.364 | 185.000 |
| $\boldsymbol{D}$ | 0.0023 | 0.0006 |

These two local optimum solutions represent the two regions, A and B, where $D > 0$ in the experimental region. Figure 6 represents Region A by displaying desirability as Block Temperature and Flow Rate vary with Flow Temperature fixed at $x_2 = 436.17$. Figure 7 represents Region B by displaying desirability as Block Temperature and Flow Rate vary with Flow Temperature fixed at $x_2 = 450$.

The figures displaying Regions A and B with Flow Temperature and Block Temperature held constant are presented in Appendix C. From all three dimensions of $x$, Region A appears larger than Region B. Region A also contains the global optimum solution.

Both del Castillo et al [9] and Kros and Mastrangelo [29] display results regarding a global maximum for this experimental region using $s_i = t_i = 1$, $i = 1, \ldots, 6$. Table 11 compares the maximum solution displayed in both of these articles.

Del Castillo et al.'s solution is displayed in the first column of Table 11. Kros and Mastrangelo's solution is in the second column; it matches del Castillo et al.'s solution closely. The third column is the solution found using the regression equations

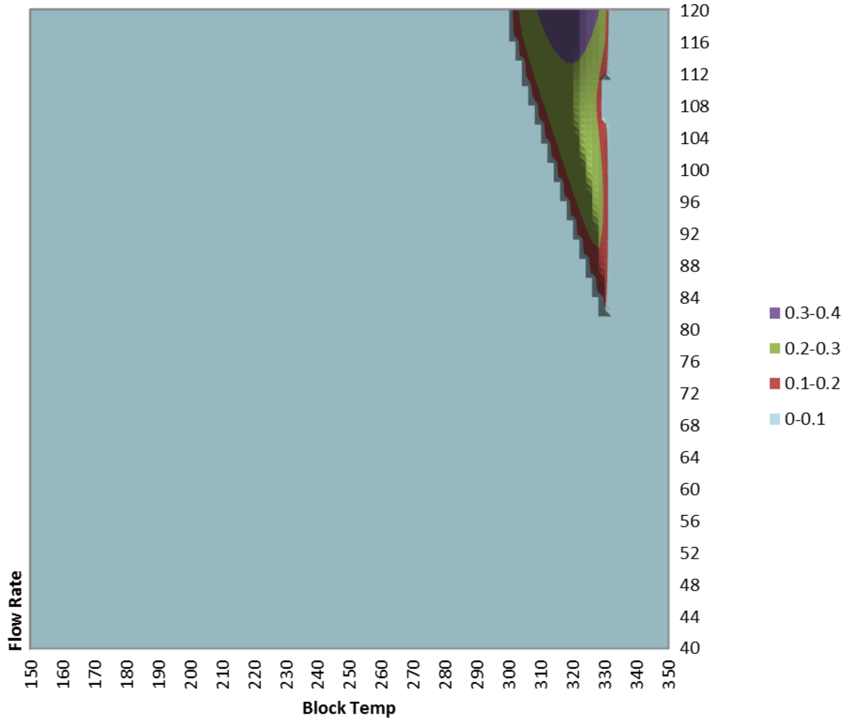


Figure 6. Value of D as Block Temp and Flow Rate vary (Flow Temp = 436.17)

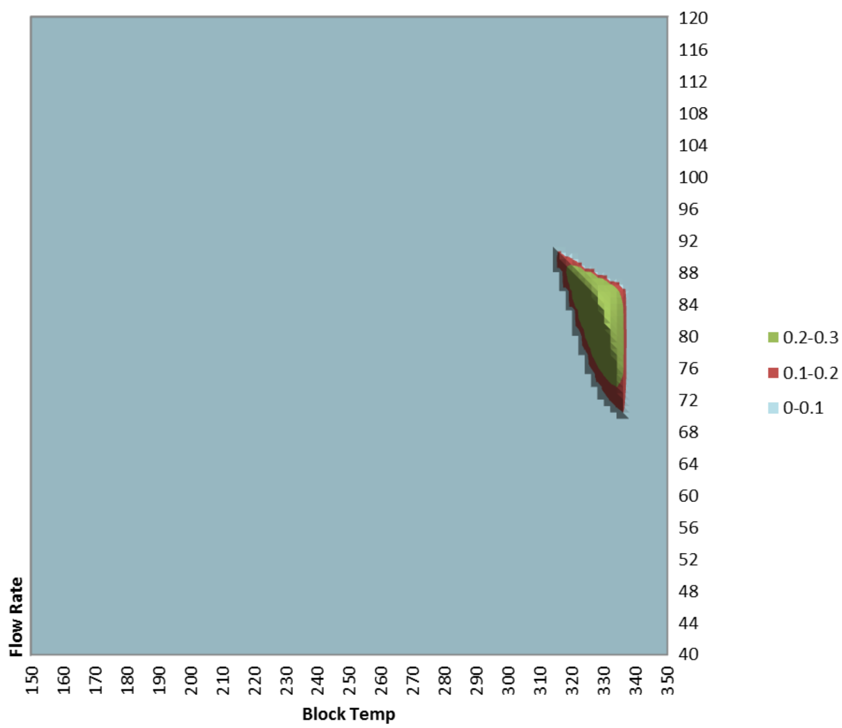


Figure 7. Value of D as Block Temp and Flow Rate vary (Flow Temp = 450)

**Table 11. Comparing global maximum found for the del Castillo example[9, 29]**

| | DC | KM | Self |
|---|---|---|---|
| $\boldsymbol{x_1}$ | 84.312 | 84.14 | 84.113 |
| $\boldsymbol{x_2}$ | 450 | 450 | 450 |
| $\boldsymbol{x_3}$ | 329.73 | 329.66 | 329.902 |
| $\boldsymbol{\hat{y}_1}$ | 186.1 | 186.06 | 186.02 |
| $\boldsymbol{\hat{y}_2}$ | 174.5 | 174.49 | 173.92 |
| $\boldsymbol{\hat{y}_3}$ | 172.1 | 172.02 | 172.06 |
| $\boldsymbol{\hat{y}_4}$ | 192.6 | 192.63 | 192.62 |
| $\boldsymbol{\hat{y}_5}$ | 173.1 | 173.04 | 173.07 |
| $\boldsymbol{\hat{y}_6}$ | 185 | 184.95 | 185.00 |
| D | 0.3076 | 0.3058 | 0.2989 |

stated by del Castillo et al. [9] in the HJ algorithm. Interestingly, this set of equations produces a global maximium in Region B instead of region A as when using Equations 4.1 through 4.6.

## 4.3 Monte Carlo Simulation

The residuals based on the above model are used to estimate the marginal distribution functions for the six responses. Appendix B contains the residuals for this regression model. The lower bound $A$ for each response's Beta distribution is chosen by rounding its lowest residual down to the next integer value. The upper bound $B$ for each response is chosen by rounding its highest residual up to the next integer value. Using these bounds, the $\alpha$ and $\beta$ parameters are estimated using Matlab's *fminsearch* function. Table 12 contains the estimated parameters of the six marginal Beta distribution functions.

**Table 12. Estimated parameters for the marginal Beta distribution functions**

| | Residual $\boldsymbol{y_1}$ | Residual $\boldsymbol{y_2}$ | Residual $\boldsymbol{y_3}$ | Residual $\boldsymbol{y_4}$ | Residual $\boldsymbol{y_5}$ | Residual $\boldsymbol{y_6}$ |
|---|---|---|---|---|---|---|
| $\alpha$ | 0.9980 | 0.9047 | 1.3739 | 0.8615 | 1.1511 | 1.1428 |
| $\beta$ | 1.1914 | 0.9218 | 1.3411 | 0.9916 | 1.5732 | 1.2991 |
| A | -15 | -11 | -16 | -9 | -5 | -5 |
| B | 31 | 21 | 30 | 18 | 11 | 10 |

The correlation matrix $\boldsymbol{R}$ is calculated from the residuals as shown in Table 13.

**Table 13. Correlation matrix of the residuals**

| | | | | | |
|---|---|---|---|---|---|
| 1 | 0.7672 | 0.6728 | 0.4184 | 0.5533 | 0.3700 |
| 0.7672 | 1 | 0.8351 | 0.6260 | 0.8313 | 0.6369 |
| 0.6728 | 0.8351 | 1 | 0.5084 | 0.6988 | 0.5849 |
| 0.4184 | 0.6260 | 0.5084 | 1 | 0.8838 | 0.9232 |
| 0.5533 | 0.8313 | 0.6988 | 0.8838 | 1 | 0.9122 |
| 0.3700 | 0.6369 | 0.5849 | 0.9232 | 0.9122 | 1 |

Using Matlab's *mvnrnd* function and the normal multivariate copula method, a set of 10,000 sample residuals are constructed for the Monte Carlo simulation. To confirm this random sample is from the intended distribution, the Beta distribution parameter estimates and correlation matrix of the sample is calculated for comparison with the residual data estimates. Table 14 contains the Beta parameters fit to the sample.

**Table 14. Beta distribution parameters fit to the Monte Carlo samples**

| | Residual $\boldsymbol{y_1}$ | Residual $\boldsymbol{y_2}$ | Residual $\boldsymbol{y_3}$ | Residual $\boldsymbol{y_4}$ | Residual $\boldsymbol{y_5}$ | Residual $\boldsymbol{y_6}$ |
|---|---|---|---|---|---|---|
| $\alpha$ | 0.9688 | 0.8878 | 1.3584 | 0.8569 | 1.1485 | 1.1336 |
| $\beta$ | 1.1618 | 0.9125 | 1.337 | 0.9789 | 1.571 | 1.2833 |

Table 15 contains the correlation matrix of the random sample. Both the parameters from the six marginal distributions and the correlation matrix from the random sample resemble those calculated from the residuals.

**Table 15. Correlation matrix for the Monte Carlo samples**

| | | | | | |
|---|---|---|---|---|---|
| 1 | 0.7545 | 0.6592 | 0.4121 | 0.5452 | 0.3660 |
| 0.7545 | 1 | 0.8211 | 0.6092 | 0.8169 | 0.6224 |
| 0.6592 | 0.8211 | 1 | 0.4859 | 0.6791 | 0.5665 |
| 0.4121 | 0.6092 | 0.4859 | 1 | 0.8712 | 0.9127 |
| 0.5452 | 0.8169 | 0.6791 | 0.8712 | 1 | 0.9056 |
| 0.3660 | 0.6224 | 0.5665 | 0.9127 | 0.9056 | 1 |

The Monte Carlo simulation is run using the random sample. First, the desirability function is treated as a utility function. The HJ algorithm is started at both of the local maxima displayed in Table 10. Risk aversion is modeled as described in Table 9. Table 16 displays the robust optimal solutions when $s_i = t_i = 3, 6, 9$ $(i = 1 \ldots, 6)$.

**Table 16. Robust optimal solutions varying by risk aversion**

| $\boldsymbol{s_i, t_i}$ | 3 | 6 | 9 |
|---|---|---|---|
| $\boldsymbol{x_1}$ | 120 | 120 | 120 |
| $\boldsymbol{x_2}$ | 407.190 | 429.415 | 431.174 |
| $\boldsymbol{x_3}$ | 349.934 | 339.771 | 337.703 |
| $\boldsymbol{E(D)}$ | 0.0212 | 0.0037 | 0.0009 |

The robust optimal solution moves slightly as the exponents vary, but it remains near Region A. The value of $E(D)$ decreases as the exponent increases due to the decreasing effect the exponent has on $d_i$. It appears, in this case, adjusting the exponents in the desirability function does not affect which area of the operating region contains the robust optimal solution when the desirability function is used as a utility function. This is most likely due to Region A containing the highest peak desirability and also being the larger region where $D > 0$.

The robust optimal solution is also calculated using the desirability function as a value function and assessing an exponential utility function over it.

$$U = 1 - e^{-\gamma D} \tag{4.7}$$

The exponents, $s_i$, $t_i$ $(i = 1, \ldots, 6)$ vary as described in Table 9 in addition to varying the risk aversion coefficient, $\gamma = 0.2, 1, 5$. Table 17 contains the set of robust optimal solutions found in this case.

The robust optimal solution moves slightly as the exponents and $\gamma$ vary, but it remains near Region A. The value of $E(U)$ decreases as the exponent increases due to the decreasing effect the exponent has on $d_i$. The value of $E(U)$ increases as $\gamma$

**Table 17. Robust optimal solutions varying $\gamma$, $s_i$, $t_i$**

| $\gamma$ | $s_i, t_i$ | 3 | 6 | 9 |
|---|---|---|---|---|
| 0.2 | $\boldsymbol{x_1}$ | 120 | 120 | 120 |
| | $\boldsymbol{x_2}$ | 407.176 | 429.413 | 431.175 |
| | $\boldsymbol{x_3}$ | 349.990 | 339.778 | 337.703 |
| | $\boldsymbol{E(U)}$ | 0.0042 | 0.0007 | 0.0002 |
| 1 | $\boldsymbol{x_1}$ | 120 | 120 | 120 |
| | $\boldsymbol{x_2}$ | 407.192 | 429.348 | 431.329 |
| | $\boldsymbol{x_3}$ | 350 | 339.854 | 337.769 |
| | $\boldsymbol{E(U)}$ | 0.0196 | 0.0035 | 0.0009 |
| 5 | $\boldsymbol{x_1}$ | 120 | 120 | 120 |
| | $\boldsymbol{x_2}$ | 407.227 | 429.018 | 431.416 |
| | $\boldsymbol{x_3}$ | 350 | 340.449 | 337.873 |
| | $\boldsymbol{E(U)}$ | 0.0739 | 0.0156 | 0.0043 |

increases due to the increasing effect it has on U. It appears, in this case, adjusting the exponents or $\gamma$ does not affect which area of the operating region contains the robust optimal solution when an exponential utility function is assessed over the desirability function. This is most likely due to Region A containing the highest peak desirability and also being the larger region where $D > 0$.

The desirabilities calculated by adding the 10,000 sample residuals to the various optimum solutions found provide insight into the robustness of the solutions. Consider the case when $s_i = t_i = 3$ ($i = 1, \ldots, 6$). Table 18 contains the average and maximum desirabilities calculated when using the deterministic solution shown in Table 10 and the robust optimal solutions shown in Table 17.

**Table 18. Maximum and average desirability for deterministic optimum and optimums at $\gamma = 0.2, 1, 5$ when $s_i = t_i = 3$**

| | **Deterministic** | $\boldsymbol{\gamma = 0.2}$ | $\boldsymbol{\gamma = 1}$ | $\boldsymbol{\gamma = 5}$ |
|---|---|---|---|---|
| $\boldsymbol{E(D)}$ | 0.0109 | 0.0212 | 0.0212 | 0.0212 |
| **Max** $\boldsymbol{D}$ | 0.5129 | 0.4112 | 0.4109 | 0.4104 |

The deterministic solution produces a higher maximum desirability when adding the set of 10,000 random residuals to it. However it produces a lower average desirability. The robust solutions found at $\gamma = 0.2, 1, 5$ produce different maximum desirabilities, however the average desirability across this set is the same.

Figure 8 displays the distribution of desirabilities across the 10,000 samples. The deterministic optimum solution is compared to the three robust solutions found with $\gamma = 0.2, 1, 5$.

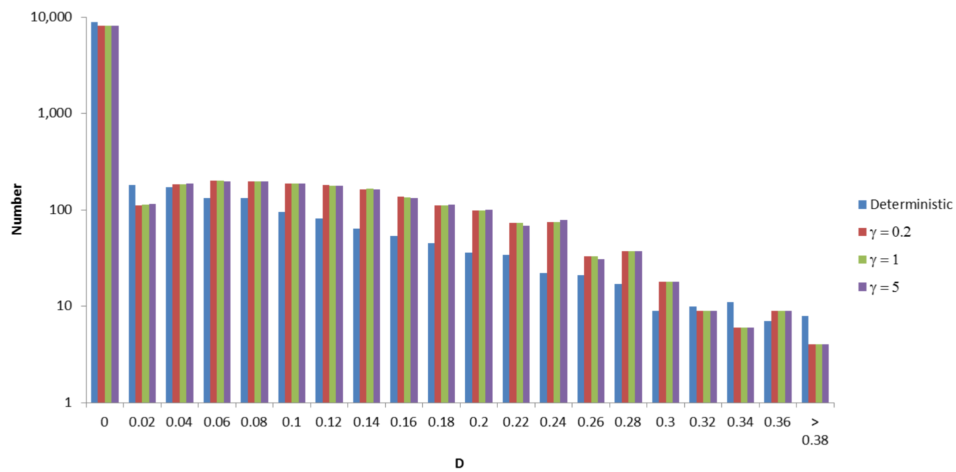


**Figure 8. Compare the distribution of $D$ between different values of $\gamma$ and the deterministic optimum when $s_i = t_i = 3$)**

The horizontal axis shows desirability. The vertical axis shows the number of samples with that particular desirability level on a logarithmic scale. The distributions of the three robust solutions are very similar. However, the deterministic solution tends to have fewer samples with higher desirabilities than the robust solutions have. In this case, a risk averse or risk neutral decision maker would prefer one of the robust solutions. A sufficiently risk seeking decision maker would prefer the deterministic solution.

Consider the exponential utility function

$$U = e^{-\gamma D} \tag{4.8}$$

where $\gamma < 0$. This utility function describes risk seeking behavior with respect to desirability and $\gamma_D^U = \gamma$. When $\gamma < -16.5$, the deterministic solution is preferred to the three robust solutions. This extreme risk seeking attitude is rarely modeled in practice.

Consider the case when $s_i = t_i = 6$ $(i = 1, \ldots, 6)$. Table 19 contains the average and maximum desirabilities calculated when using the deterministic solution shown in Table 10 and the robust optimal solutions shown in Table 17.

**Table 19. Maximum and average desirability for deterministic optimum and optimums at $\gamma = 0.2, 1, 5$ when $s_i = t_i = 6$**

| | Deterministic | $\gamma = 0.2$ | $\gamma = 1$ | $\gamma = 5$ |
|---|---|---|---|---|
| $E(D)$ | 0.0018 | 0.0037 | 0.0037 | 0.0037 |
| Max $D$ | 0.2631 | 0.2873 | 0.2884 | 0.2893 |

The deterministic solution produces a lower maximum desirability and lower average desirability when adding the set of 10,000 random residuals to it. The robust solutions found at $\gamma = 0.2, 1, 5$ produce different maximum desirabilities, however the average desirability across this set is the same.

Figure 9 displays the distribution of desirabilities across the 10,000 samples. The deterministic optimum solution is compared to the three robust solutions found with $\gamma = 0.2, 1, 5$.

The horizontal axis shows desirability. The vertical axis shows number of samples with that particular desirability level on a logarithmic scale. The distributions of the three robust solutions are very similar. However, the deterministic solution tends to have fewer samples with higher desirabilities than the robust solutions have. In this case, since the robust solutions had the highest maximum desirability and

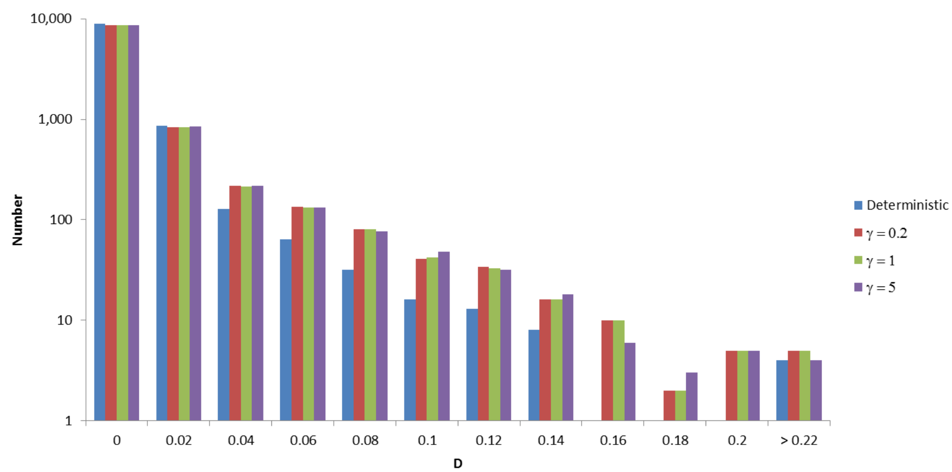


**Figure 9. Compare the distribution of $D$ between different values of $\gamma$ and the deterministic optimum when $s_i = t_i = 6$)**

highest average desirability, a decision maker would prefer a robust solution over the deterministic solution regardless of risk preference.

Consider the case when $s_i = t_i = 9$ $(i = 1, \ldots, 6)$. Table 20 contains the average and maximum desirabilities calculated when using the deterministic solution shown in Table 10 and the robust optimal solutions shown in Table 17.

**Table 20. Maximum and average desirability for deterministic optimum and optimums at $\gamma = 0.2, 1, 5$ when $s_i = t_i = 9$**

| | **Deterministic** | $\boldsymbol{\gamma = 0.2}$ | $\boldsymbol{\gamma = 1}$ | $\boldsymbol{\gamma = 5}$ |
|---|---|---|---|---|
| $\boldsymbol{E(D)}$ | 0.0004 | 0.0009 | 0.0009 | 0.0009 |
| **Max** $\boldsymbol{D}$ | 0.1350 | 0.1488 | 0.1462 | 0.1434 |

The deterministic solution produces a lower maximum desirability and lower average desirability when adding the set of 10,000 random residuals to it. The robust solutions found at $\gamma = 0.2, 1, 5$ produce different maximum desirabilities, however the average desirability across this set is the same.

Figure 10 displays the distribution of desirabilities across the 10,000 samples. The deterministic optimum solution is compared to the three robust solutions found with $\gamma = 0.2, 1, 5$.

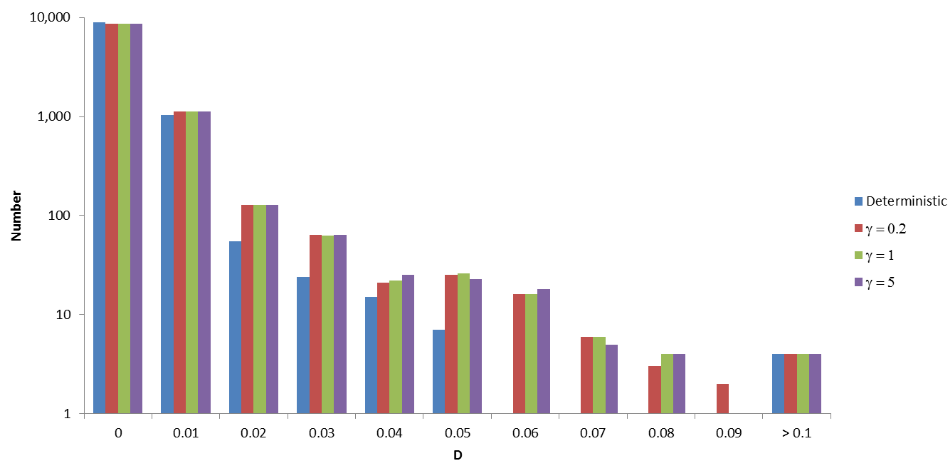


**Figure 10. Compare the distribution of $D$ between different values of $\gamma$ and the deterministic optimum when $s_i = t_i = 9$)**

The horizontal axis shows desirability. The vertical axis shows number of samples with that particular desirability level on a logarithmic scale. The distributions of the three robust solutions are very similar. However, the deterministic solution tends to have fewer samples with higher desirabilities than the robust solutions have. In this case, since the robust solutions had the highest maximum desirability and highest average desirability, a decision maker would prefer a robust solution over the deterministic solution regardless of risk preference.

# V. Robust Stability Operations Policy Findings

## 5.1 National Operational Environment Model Experiment

To illustrate the decision analysis approach to robust optimization, a stability operations policy optimization problem is investigated within the Air Force Research Laboratory's (AFRL) National Operational Environment Model (NOEM).

An experiment is designed to investigate how various diplomatic, informational, military and economic (DIME) instruments of national power affect the Democratic Republic of Congo's (DRC) political, military, economic, social, infrastructure, and information (PMESII) systems.

This experiment investigates the effect 14 DIME factors have on two PMESII responses. Table 21 lists the factors with brief descriptions. Table 22 lists the units and minimum and maximum values for the experimental factors.

National debt and total number of activists are the two indicators of the DRC's PMESII systems. Minimizing both responses is preferred and is indicative of a more stable government.

Each design point is run for three simulated years (1095 days) with two replications. A list of random seed generators is generated in Excel and each design point is assigned a seed from this list. NOEM outputs debt and activist data for every simulated day. This experiment considers the arithmetic mean of the last 30 days of the simulation's debt and activist output as the two responses for debt and activists respectively.

**Table 21. NOEM DRC experiment factors and descriptions**

| Factor | Description |
|---|---|
| **Diplomatic** | |
| Stimulus maximum | Maximum amount of money allocated daily to government stimulus |
| Stimulus Spending % | Percentage of government funding earmarked for stimulus |
| Government Corruption | Proportion of government income not available for use |
| **Military** | |
| Initial Police Forces | Police forces at the start of the simulation |
| Police Forces Goal | Long term goal for police forces |
| Jail Term | Mean jail term for arrested activist |
| Mean Adjudication Processing Time | Mean time spend in adjudication process |
| Adjudication Rate | Average rate of adjudication process |
| **Economic** | |
| Tax Rate | Income and production tax rate |
| Interest Rate | Government debt interest rate |
| Long Term Government Employee Share | Long term proportion of workers employed by the government |
| Government Wages | Mean annual wage paid to government employees |
| Infrastructure Spending % | Percentage of government funding earmarked for infrastructure |
| Services Spending % | Percentage of government funding earmarked for providing services |

Table 22. **NOEM DRC experiment factors, units, minimum values, maximum values**

| Factor | Units | Min | Max |
|---|---|---|---|
| **Diplomatic** | | | |
| Stimulus maximum | \$ | 0 | 6,000,000 |
| Stimulus Spending % | % | 0 | 100 |
| Government Corruption | n/a | 0.05 | 0.15 |
| **Military** | | | |
| Initial Police Forces | personnel | 52,500 | 137,500 |
| Police Forces Goal | personnel | 112,500 | 187,500 |
| Jail Term | days | 3 | 100 |
| Mean Adjudication Processing Time | day | 0.1 | 0.9 |
| Adjudication Rate | per day | 0.01 | 0.13 |
| **Economic** | | | |
| Tax Rate | n/a | 0.03 | 0.5 |
| Interest Rate | n/a | 0.0225 | 0.0675 |
| Long Term Government Employee Share | n/a | 0.001 | 0.05 |
| Government Wages | \$ | 1400 | 4200 |
| Infrastructure Spending % | % | 0 | 100 |
| Services Spending % | % | 0 | 100 |

The experiment executes a D-optimal design. D-optimality focuses on good model coefficient estimation. It does this by choosing design points so that the determinant of the moment matrix $\boldsymbol{M}$ is maximized [32].

$$|\boldsymbol{M}| = \frac{|\boldsymbol{X'X}|}{N^p} \tag{5.1}$$

where $\boldsymbol{X}$ is the design matrix, $N$ is the number of experiment runs and $p$ is the number of parameters [32]. An experimental design consisting of 139 design points with two replicates each is chosen to create a full quadratic model with cubic terms.

A deterministic value function, $V$, is formulated to describe the preferred relationship between the two responses.

$$V = 18.164 - 5.75 \times 10^{-10} y_1 - 1.01 \times 10^{-4} y_2, \tag{5.2}$$

where $y_1$ is debt in dollars, and $y_2$ is the number of activists.

$$t(y_2, y_1) = \frac{V'_{y_2}}{V'_{y_1}} = 176,439.50 \tag{5.3}$$

The value function $V$ is describing a tradeoff where the decision maker would be willing to increase the debt by \$176,439.50 in order to reduce activists by one.

This particular form of a value function assumes a constant trade-off between the two responses throughout the entire response space. Consider two examples, one with \$3 billion of debt and 500 activists, and a second with \$3 billion of debt and 50,000 activists. It is unlikely a decision maker would have the same tradeoff between debt and number of activists in these two cases. Given \$3 billion of debt, a decision maker would more likely have a higher tradeoff when faced with 50,000 activists as opposed to when faced with 500 activists. Moreover, given a particular number of activists, a decision maker would more likely have a decreasing tradeoff as debt increases.

Consider an exponential utility function assessed over $V$

$$U = \begin{cases} 1 + e^{-\gamma V}, & \gamma < 0 \\ V, & \gamma = 0 \\ 1 - e^{-\gamma V}, & \gamma > 0 \end{cases} . \tag{5.4}$$

After assessing a utility function over value, the risk aversion with respect to the two attributes can be calculated. The objective in this experiment is to maximize V which minimizes $y_1$ and $y_2$. Since this is a decreasing value function, the risk aversion functions with respect to these attributes change some of the signs in Equations 2.11 and 2.10.

$$\gamma_{y_i}^V = -\frac{V''_{y_i}}{V'_{y_i}},\ i = 1, 2 \tag{5.5}$$

$$\gamma_{y_i}^U = -\gamma_V^U V'_{y_i} + \gamma_{y_i}^V,\ i = 1, 2 \tag{5.6}$$

This value function is additive, so its contribution to risk aversion is zero.

$$\gamma_{y_i}^V = 0,\ i = 1, 2 \tag{5.7}$$

The risk aversion of the utility function with respect to the two attributes are

$$\gamma_{y_1}^U = 5.75 \times 10^{-10}\gamma \text{ and} \tag{5.8}$$

$$\gamma_{y_2}^U = 1.01 \times 10^{-4}\gamma. \tag{5.9}$$

In cases where constant risk aversion is indicated by the utility function, one must be cognizant of saturation effect. The utility function, $U = 1 - e^{-\gamma V}$, equals one as $V$ approaches positive utility. When $V$ is greater than $\approx 1/\gamma$, the utility function is nearly a horizontal line. This saturation effect improperly models risk preference at these higher levels of $V$. This limitation of the exponential utility function must be considered when selecting reasonable values of $\gamma$.

## 5.2 Experiment Results

After the experiment is run, the data is analyzed in Design-Expert. Ordinary least squares (OLS) regression is applied and two regression equations to predict $y_1$ and $y_2$ are developed.

The model for $y_1$ did not pass the lack-of-fit test. However the R-Squared, adjusted R-Squared and predicted R-Squared measures are all high so this model is accepted. After a natural log transform is applied to the $y_2$ response this model does pass the lack-of-fit test. Its R-Squared, adjusted R-Squared and predicted R-Squared are also high. These results are displayed in Table 23. The coefficients for these two functions appear in Appendix D.

**Table 23. Statistics for the two regression equations**

| | $y_1$ | $\ln y_2$ |
|---|---|---|
| R-Squared | 0.9882 | 0.9920 |
| Adj R-Squared | 0.9773 | 0.9847 |
| Pred R-Squared | 0.9631 | 0.9686 |

### 5.3 Deterministic optimum setting

The Hooke-Jeeves (HJ) algorithm finds the maximum value and corresponding operating solution. One thousand random starting points within the experimental region are generated using the uniform psuedo-random number generator in Microsoft Excel Visual Basic for Applications (VBA). This procedure produces 657 local maxima.

The set of local maxima is reduced using a K means clustering algorithm with Matlab's *kmeans* function. This algorithm classifies the 657 points into $K$ groups by minimizing the sum of squares of Euclidean distances between the points and their corresponding cluster centroid [30].

To find the proper number of clusters $K$, $K$ is chosen to vary between 2 and 50 and the total sum of squared distances in each case is graphed against $K$ to find a point where the trade-off between the sum and $K$ is balanced. Figure 11 displays this graph.

Based on this chart, 13 clusters are chosen which gives a total sum of squared distances of 788.05. The maximum value point from each cluster is chosen to represent

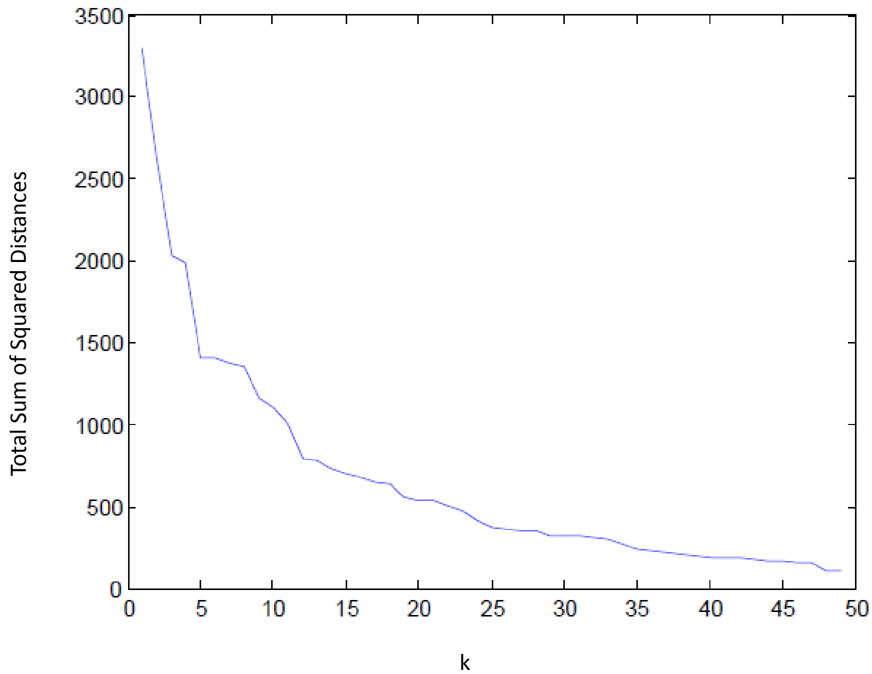


**Figure 11. Total Sum of Squared Distances in all $K$ clusters by $K$**

its cluster as a starting point for the Monte Carlo simulation. These points are displayed in Table 24.

**Table 24. Local deterministic optimum policies**

| Adjud Proc Time $x_1$ | Infra Spend $x_2$ | Govt Employ $x_3$ | Interest Rate $x_4$ | Police Goal $x_5$ | Tax Rate $x_6$ | Adjud Rate $x_7$ | Stim Pct $x_8$ | Govt Wage $x_9$ | Service Spend $x_{10}$ | Corrupt Pct $x_{11}$ | Police Init $x_{12}$ | Jail Term $x_{13}$ | Stim $x_{14}$ | V |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.9 | 100 | 0.05 | 0.0675 | 165611 | 0.480 | 0.130 | 100 | 4188.61 | 0 | 0.141 | 52500 | 87 | 0 | 15.826 |
| 0.9 | 100 | 0.001 | 0.0225 | 164944 | 0.480 | 0.110 | 100 | 4200.00 | 0 | 0.130 | 52500 | 100 | 0 | 14.501 |
| 0.1 | 4.22 | 0.001 | 0.0225 | 166091 | 0.386 | 0.010 | 0 | 1400.00 | 0 | 0.050 | 137500 | 79 | 6000000 | 14.208 |
| 0.9 | 1.54 | 0.005 | 0.0225 | 166388 | 0.480 | 0.089 | 100 | 4200.00 | 0 | 0.125 | 137500 | 98 | 0 | 14.131 |
| 0.9 | 10.11 | 0.001 | 0.0225 | 167133 | 0.461 | 0.116 | 0 | 3985.57 | 0 | 0.050 | 137500 | 88 | 0 | 14.067 |
| 0.9 | 100 | 0.004 | 0.0225 | 162504 | 0.480 | 0.110 | 100 | 4200.00 | 0 | 0.130 | 137500 | 96 | 0 | 13.957 |
| 0.9 | 4.87 | 0.001 | 0.0225 | 169242 | 0.458 | 0.108 | 0 | 1400.00 | 0 | 0.050 | 137500 | 88 | 0 | 13.833 |
| 0.9 | 100 | 0.05 | 0.0675 | 169510 | 0.465 | 0.130 | 100 | 1400.00 | 0 | 0.138 | 52500 | 88 | 0 | 13.215 |
| 0.9 | 6.41 | 0.001 | 0.0225 | 167085 | 0.422 | 0.090 | 0 | 1400.00 | 0 | 0.050 | 137500 | 83 | 6000000 | 12.623 |
| 0.9 | 14.33 | 0.001 | 0.0225 | 166446 | 0.461 | 0.118 | 0 | 3970.42 | 0 | 0.050 | 52500 | 92 | 0 | 11.950 |
| 0.9 | 100 | 0.001 | 0.0225 | 157331 | 0.470 | 0.110 | 0 | 4200.00 | 0 | 0.118 | 137500 | 90 | 0 | 11.490 |
| 0.9 | 100 | 0.001 | 0.0225 | 153712 | 0.439 | 0.093 | 0 | 4101.30 | 0 | 0.123 | 137500 | 82 | 6000000 | 10.514 |
| 0.9 | 100 | 0.05 | 0.0675 | 155220 | 0.383 | 0.130 | 0 | 1400.00 | 80.11 | 0.129 | 52500 | 65 | 6000000 | 5.090 |

## 5.4 Monte Carlo Simulation

The residuals from the regression equations are used to estimate the marginal distribution functions for the two responses. Appendix D contains the residuals for this regression model. The lower bound $A$ for each response's Beta distribution is chosen by rounding its lowest residual down to the next integer value. The upper bound $B$ for each response is chosen by rounding its highest residual up to the next integer value. Using these bounds, the $\alpha$ and $\beta$ parameters are estimated using Matlab's *fminsearch* function. Table 25 contains the estimated parameters of the two marginal Beta distribution functions.

**Table 25. Estimated parameters for the marginal Beta distribution functions**

| | **Residual $y_1$** | **Residual $\ln y_2$** |
|---|---|---|
| $\alpha$ | 17.7591 | 255.1359 |
| $\beta$ | 7.9396 | 255.7977 |
| A | -4.422E+09 | -3 |
| B | 6.389E+09 | 6 |

The correlation matrix $\boldsymbol{R}$ is calculated from the residuals as shown in Table 26.

**Table 26. Correlation matrix of the residuals**

| | |
|---|---|
| 1 | -0.1912 |
| -0.1912 | 1 |

Using Matlab's *mvnrnd* function and the normal multivariate copula method, a set of 10,000 sample residuals are constructed for the Monte Carlo simulation. To confirm this random sample is from the intended distribution, the Beta distribution parameter estimates and correlation matrix of the sample is calculated for comparison with the residual data estimates. Table 27 contains the Beta parameters fit to the sample.

**Table 27. Beta distribution parameters fit to the Monte Carlo samples**

| | **Residual** $y_1$ | **Residual** $\ln \boldsymbol{y_2}$ |
|---|---|---|
| $\alpha$ | 18.0711 | 260.9499 |
| $\beta$ | 8.0698 | 261.6797 |

Table 28 contains the correlation matrix of the random sample. Both the parameters from the two marginal distributions and the correlation matrix from the random sample appear to resemble those calculated from the residuals.

**Table 28. Correlation matrix for the Monte Carlo samples**

| | |
|---|---|
| 1 | -0.1833 |
| -0.1833 | 1 |

The Monte Carlo simulation is run using the random sample. In this model, the regression equation for $y_2$ contains a natural log transformation on the response. The random sample is not simply added to the responses. The Monte Carlo simulation allows one to compute the expected utility for a particular vector $\boldsymbol{x}$. Any point, $\boldsymbol{x}*$ gives an expected utility,

$$\sum_{i=1}^{n} \left( \frac{1}{n} U_V \left( V \left( f_1(\boldsymbol{x}*) + \epsilon_{1i}, e^{\ln(f_2(\boldsymbol{x}*))+\epsilon_{2i}} \right) \right) \right) \tag{5.10}$$

where $f_i(\boldsymbol{x}*)$ is the expected response $y_i$ given an input of $\boldsymbol{x}*$ $(i = 1, 2)$, $n$ is the number of random samples used in the Monte Carlo simulation, $\epsilon_{1i}$ and $\epsilon_{2i}$ are the $i$-th random samples from the distributions of the residuals for $y_1$ and $y_2$ respectively, $V$ is the multiattribute value function, and $U_V$ is the utility function assessed over value. The optimization problem is to then choose a point $\boldsymbol{x}*$ that maximizes expected utility.

The HJ algorithm is started at each local maxima displayed in Table 24. Risk preferences are modeled by varying $\gamma$ in the appropriate utility function in Equation

5.4. Table 29 contains the levels of $\gamma$ chosen and the corresponding risk preference with respect to $y_1$ and $y_2$.

**Table 29. Levels of $\gamma$ sampled in Monte Carlo simulation**

| $\gamma$ | $\gamma^U_{y_1}$ | $\gamma^U_{y_2}$ |
|---|---|---|
| -0.01 | -5.751E-12 | -1.015E-06 |
| 0 | 0 | 0 |
| 0.01 | 5.751E-12 | 1.015E-06 |
| 0.02 | 1.150E-11 | 2.030E-06 |

Table 30 displays the robust solutions found when $\gamma = -0.01, 0, 0.01, 0.02$ alongside the deterministic solution. The robust solutions when $\gamma = 0, 0.01, 0.02$ are equal. They differ slightly from the deterministic solution and the robust solution when risk seeking behavior is modeled ($\gamma = -0.01$).

**Table 30. Compare deterministic solution with robust solutions for $\gamma = -0.01, 0, 0.01, 0.02$**

| | **Deterministic Solution** | **Robust Solutions** $\gamma =$ | |
|---|---|---|---|
| | | $\mathbf{-0.01}$ | $\mathbf{0, 0.01, 0.02}$ |
| $x_1$ | 0.9 | 0.9 | 0.9 |
| $x_2$ | 100 | 100 | 100 |
| $x_3$ | 0.05 | 0.05 | 0.05 |
| $x_4$ | 0.0675 | 0.0675 | 0.0675 |
| $x_5$ | 165481 | 165473 | 165473 |
| $x_6$ | 0.47811 | 0.47814 | 0.47806 |
| $x_7$ | 0.13 | 0.13 | 0.13 |
| $x_8$ | 100 | 100 | 100 |
| $x_9$ | 4188.76 | 4188.84 | 4188.84 |
| $x_{10}$ | 0 | 0 | 0 |
| $x_{11}$ | 0.14081 | 0.14081 | 0.14081 |
| $x_{12}$ | 52500 | 52500 | 52500 |
| $x_{13}$ | 87 | 87 | 87 |
| $x_{14}$ | 0 | 0 | 0 |
| $E(V)$ | 15.8259 | 15.8259 | 15.8259 |

This case illustrates a limitation in this method of finding a robust optimum solution. The methodology assumes constant variance in the response noise over the experimental region. The Monte Carlo simulation inputs noise to the responses,

$y_1, \ldots, y_n$, and calculates a robust solution by examining how this noise affects the multiattribute value function. To illustrate this, consider the example robust solution in Chapter 2, redisplayed in Figure 12.

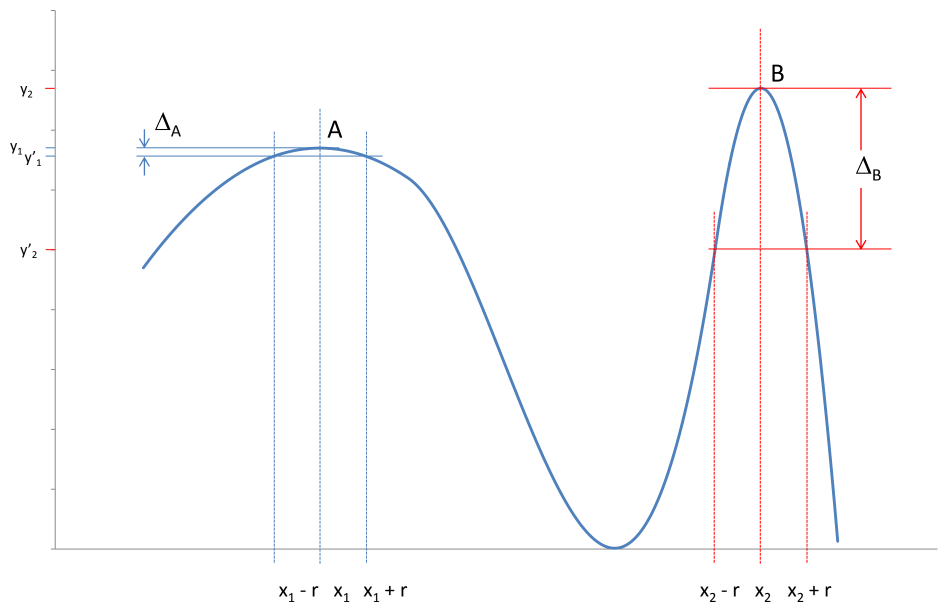


**Figure 12. Illustration of Robust Solution [16]**

Here, the horizontal axis represents the responses $(y_1, \ldots, y_n)$ and the vertical axis represents $V(y_1, \ldots, y_n)$. By properly specifying the decision maker's risk preference in the utility function $U_V(V(y_1, \ldots, y_n))$, the proper robust solution can be found.

This method is dependent on the form of the value function in use. The value function illustrated in Figure 12 has curvature present. The value function used in this situation is an additive value function. It exhibits no curvature. Therefore this method will choose a robust solution at or near the optimum.

These three points are input into NOEM with 10 replicates run for each point to test the prediction of these optimum solutions. Table 31 contains the expected values and 95% confidence intervals for these three points.

**Table 31. Average $V$ and 95% confidence intervals from 10 replicates in NOEM for the three optimum points**

| | | Robust Solution $\gamma =$ | |
|---|---|---|---|
| | **Deterministic** | $\mathbf{-0.01}$ | $\mathbf{0, 0.01, 0.02}$ |
| **E(V)** | 0.6577 | 0.7156 | 0.1472 |
| **95% Conf Int** | [0.5837, 0.7316] | [0.6073, 0.8238] | [-0.1963, 0.4907] |

The expected values ($E(V)$) displayed in Table 30 do not fall within the 95% confidence intervals found by running these points in NOEM. This is most likely due to no design point from the experiment lies near these calculated optima. A space-filling design such as a Latin hypercube design may present better results due to the experiment design points being more uniformly spread throughout the experimental region. Augmenting this D-optimal design with additional runs may also have presented a better model of the response surface. Moreover, a Kriging model of the system would most likely have described the surface better than a regression model. A Kriging model makes no assumption about the form of the underlying system response. Moreover, applications of Kriging models exist that model either variance homogeneity or variance heterogeneity [26].

# VI. Summary, Conclusions and Recommendations

Decisions considering a set of multiple objectives occur everyday in government, military, and industrial settings. After understanding the decision maker's preferences regarding the trade-offs between the objectives and risk attitude, a utility function can be constructed to provide insight to the decision maker regarding such a decision.

When the decision situation concerns a system containing multiple inputs and outputs, response surface methodology provides a means to model the system with a set of equations. This set of equations can be used to inform the determination of value and utility functions that best describe the decision situation.

Many systems contain noise that effect the system outputs. In these cases, the best solution may not be simply the solution with the optimum expected output. The best solution may be in an operating location where the output varies less in the presence of noise. Depending on the form of the value function, this robust solution can be modeled and found by properly describing the risk attitude of the decision maker in a Monte Carlo simulation.

The desirability function has emerged as a popular method of scalarizing a multiple response optimization problem. It is a powerful function when the parameters are chosen properly to describe the decision maker's preferences.

In the case of the maximum is better desirability function, the lower bounds and target levels of the responses must be carefully chosen. A lower bound that is too high may eliminate desirable solutions. A target level that is too low may value multiple responses equally when one is more desirable.

When the desirability function is treated as a value function, the deterministic trade-offs described between responses are dependent upon the responses and their corresponding exponents and lower bounds. Trade-offs are not constant as Kros and

Mastrangelo calculated [29]. Once the lower bounds and targets are set, the exponents can be adjusted to properly describe the decision maker's trade-off preferences.

When a utility function is assessed over desirability, the risk preferences described depend on the form of the utility function. In the case of an exponential utility function, the risk preference with respect to the responses depends on the responses and their respective lower bounds, target levels, and desirability function exponents. Adjusting the lower bound or exponent has a non-monotonic effect on risk preference and must be adjusted carefully.

When the desirability function is treated as a utility function, there exists an implicit assumption of utility independence between all responses. The risk preference with respect to each response depends on the response, its exponent, and lower bound. The desirability function can describe risk averse, risk neutral, and risk seeking behavior depending on whether the exponent is less than, equal to, or greater than the number of responses. This is in contrast to Kros and Mastrangelo stating the risk preference described by the desirability function depends on whether the exponent is less than, equal to, or greater than one [29].

A decision analysis method of calculating a robust optimum solution using a utility function assigned over a value function in a Monte Carlo simulation is applied to a wire-bonding process experiment from the quality and reliability engineering design literature and to a national policy-making scenario within NOEM.

The usefulness of this method depends on the form of the value function. Since random noise is assumed to be constant throughout the experimental region, certain value functions (e.g., an additive value function) are affected uniformly by this random noise. When the value function exhibits curvature (e.g., the desirability function), the effect noise has on the variance in the value function can be measured and a robust optimum solution can be found that is affected by changing the risk preference described by the utility function.

Additional research in this area includes analyzing other multiple response optimization functions. Any multiple response optimization function makes implicit and explicit assumptions regarding the decision maker's preferences. These assumptions should be properly analyzed so that the decision maker can make informed decisions.

The methodology presented for finding a robust optimum solution presented in this thesis provides insight in only certain forms of the value function. It does not provide additional insight about robustness when an additive value function is used. It can provide insight into a situation when a desirability function is used as the value function. Applying this method across a larger set of value functions should provide a fuller understanding about the types of functions for which this method provides insight about robustness.

The methodology presented here assumes random noise with constant variance throughout the experimental region. Further research can include modeling heterogeneous variance and applying a dynamic random sample that changes relative to the variance modeled at the current point in the experimental region within a Monte Carlo simulation.

The Kriging method can also estimate the response surface assuming either homogeneous or heterogeneous variance within the system. Further research includes how best to construct a Monte Carlo simulation that models the variance assumed in a Kriging model of the system.

In the current culture of constrained budgets, decision makers are faced more than ever with making decisions that encompass multiple, competing objectives. A method that properly describes the decision maker's preferences regarding risk and the tradeoffs between the objectives at hand while also taking uncertainty into account will serve to provide useful insight into the decision at hand. With that insight, the decision maker can proceed with confidence that he or she has made the best decision given current information.

## Appendix A. Desirability function analysis in $n$-dimensions

Consider the desirability function in $n$-dimensions.

$$D = (d_1 \cdots d_n)^{\frac{1}{n}} \tag{A.1}$$

Consider a system with $m$ inputs, $(x_1, \ldots, x_m)$. Each input, $x_i$ falls within the experimental region defined by $x_i \in [x_i^0, x_i^*]$, $i = 1, \ldots, m$. These inputs cause changes in two responses, $y_1$ and $y_2$ which each fall within the range, $y_i \in [y_i^0, y_i^*]$, $i = 1, \ldots, n$. This analysis focuses on the case where a maximum response is desired as displayed in Equation 2.21 with no loss of generality since the other forms of the desirability function can be transformed into this case. Only the non-trivial piece of Equation 2.21 where $L_i < y_i < T_i, i = 1, \ldots, n$ is considered. Equation A.2 displays this non-trivial piece.

$$d_i = \left[\frac{y_i - L_i}{T_i - L_i}\right]^{r_i}, L_i < y_i < T_i \tag{A.2}$$

Where $[L_i, T_i] \subseteq [y_i^0, y_i^*]$, $i = 1, \ldots, n$. This forces $0 < D < 1$.

The partial derivative of $D$ with respect to $y_i$, $(1 \leq i \leq n)$ is

$$D'_{y_i} = \frac{r_i}{n(T_i - L_i)} \left[\frac{y_1 - L_1}{T_1 - L_1}\right]^{\frac{r_1}{n}} \cdots \left[\frac{y_i - L_i}{T_i - L_i}\right]^{\frac{r_i}{n} - 1} \cdots \left[\frac{y_n - L_n}{T_n - L_n}\right]^{\frac{r_n}{n}} \tag{A.3}$$

The tradeoff function between $y_i$ and $y_j$ $(i \neq j)$ is defined as

$$t(y_i, y_j) = \frac{D'_{y_i}}{D'_{y_j}} = -\frac{dy}{dx}|_{\text{isopreference contour}}. \tag{A.4}$$

The deterministic tradeoffs between the two attributes can then be stated.

$$t(y_i, y_j) = \frac{r_i}{r_j}\left[\frac{y_j - L_j}{y_i - L_i}\right] \tag{A.5}$$

Consider assessing an exponential utility function over $D$.

$$U = 1 - e^{-\gamma D} \tag{A.6}$$

The risk aversion of the utility function with respect to $y_i$ $(1 \leq i \leq n)$ is

$$\gamma^U_{y_i} = \gamma^U_D D'_{y_i} + \gamma^D_{y_i}. \tag{A.7}$$

The risk aversion for the utility function with respect to the desirability function is simply

$$\gamma^U_D = \gamma. \tag{A.8}$$

The contribution of the desirability function to the risk aversion of the attribute $y_i$ is

$$\gamma^D_{y_i} = \frac{1 - \frac{r_i}{n}}{y_i - L_i}. \tag{A.9}$$

The risk aversion expressed by this utility function with respect to $y_i$ is

$$\gamma^U_{y_i} = \frac{\gamma r_i}{n(y_i - L_i)}\left[\frac{y_1 - L_1}{T_1 - L_1}\right]^{\frac{r_1}{n}} \cdots \left[\frac{y_i - L_i}{T_i - L_i}\right]^{\frac{r_i}{n}} \cdots \left[\frac{y_n - L_n}{T_n - L_n}\right]^{\frac{r_n}{n}} + \frac{1 - \frac{r_i}{n}}{y_i - L_i}. \tag{A.10}$$

# Appendix B. Data from del Castillo example

**Table 32. Data table from wire bonding experiment [9]**

| n | $x_1$ Flow Rate | $x_2$ Flow Temp | $x_3$ Block Temp | $y_1$ Max Temp A | $y_2$ Begin Bond A | $y_3$ Finish Bond A | $y_4$ Max Temp B | $y_5$ Begin Bond B | $y_6$ Finish Bond B |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 40 | 200 | 250 | 139 | 103 | 110 | 110 | 113 | 126 |
| 2 | 120 | 200 | 250 | 140 | 125 | 126 | 117 | 114 | 131 |
| 3 | 40 | 450 | 250 | 184 | 151 | 133 | 147 | 140 | 147 |
| 4 | 120 | 450 | 250 | 210 | 176 | 169 | 199 | 169 | 171 |
| 5 | 40 | 325 | 150 | 182 | 130 | 122 | 134 | 118 | 115 |
| 6 | 120 | 325 | 150 | 170 | 130 | 122 | 134 | 118 | 115 |
| 7 | 40 | 325 | 350 | 175 | 151 | 153 | 143 | 146 | 164 |
| 8 | 120 | 325 | 350 | 180 | 152 | 154 | 152 | 150 | 171 |
| 9 | 80 | 200 | 150 | 132 | 108 | 103 | 111 | 101 | 101 |
| 10 | 80 | 450 | 150 | 206 | 143 | 138 | 176 | 141 | 135 |
| 11 | 80 | 200 | 350 | 183 | 141 | 157 | 131 | 139 | 160 |
| 12 | 80 | 450 | 350 | 181 | 180 | 184 | 192 | 175 | 190 |
| 13 | 80 | 325 | 250 | 172 | 135 | 133 | 155 | 138 | 145 |
| 14 | 80 | 325 | 250 | 190 | 149 | 145 | 161 | 141 | 149 |
| 15 | 80 | 325 | 250 | 180 | 141 | 139 | 158 | 140 | 148 |

**Table 33. Residuals from del Castillo example experiment**

| n | Residual $y_1$ | Residual $y_2$ | Residual $y_3$ | Residual $y_4$ | Residual $y_5$ | Residual $y_6$ |
|---|---|---|---|---|---|---|
| 1 | -12.558 | -10.375 | -6.575 | -4.125 | -3.5 | -1.125 |
| 2 | -11.558 | -0.375 | -3.825 | 8.375 | 3 | 4.375 |
| 3 | -14.308 | -5.625 | -15.575 | -5.875 | -2 | -1.875 |
| 4 | 11.692 | 7.375 | 7.175 | 6.625 | 4.5 | 3.625 |
| 5 | 10.692 | 9.125 | 9.8 | 8.375 | 5.25 | 4.375 |
| 6 | -1.308 | -2.875 | -3.45 | -8.625 | -3.25 | -4.625 |
| 7 | -3.558 | 1.875 | 0.05 | 1.625 | 0.25 | -1.375 |
| 8 | 1.442 | -9.125 | -12.2 | -6.375 | -4.25 | -3.375 |
| 9 | 3.067 | 2.75 | 0.175 | -5.357 | -2.036 | -2.857 |
| 10 | -7.683 | -5.5 | 3.175 | -1.607 | -1.536 | -0.107 |
| 11 | 8.817 | 7.5 | 13.425 | -1.107 | 2.964 | 1.393 |
| 12 | -1.933 | 3.25 | 8.425 | -1.357 | -0.536 | 0.143 |
| 13 | -2.933 | -6 | -6.2 | 0.143 | -1.286 | -1.857 |
| 14 | 15.067 | 8 | 5.8 | 6.143 | 1.714 | 2.143 |
| 15 | 5.067 | 0 | -0.2 | 3.143 | 0.714 | 1.143 |

# Appendix C. Feasible Region of del Castillo experiment

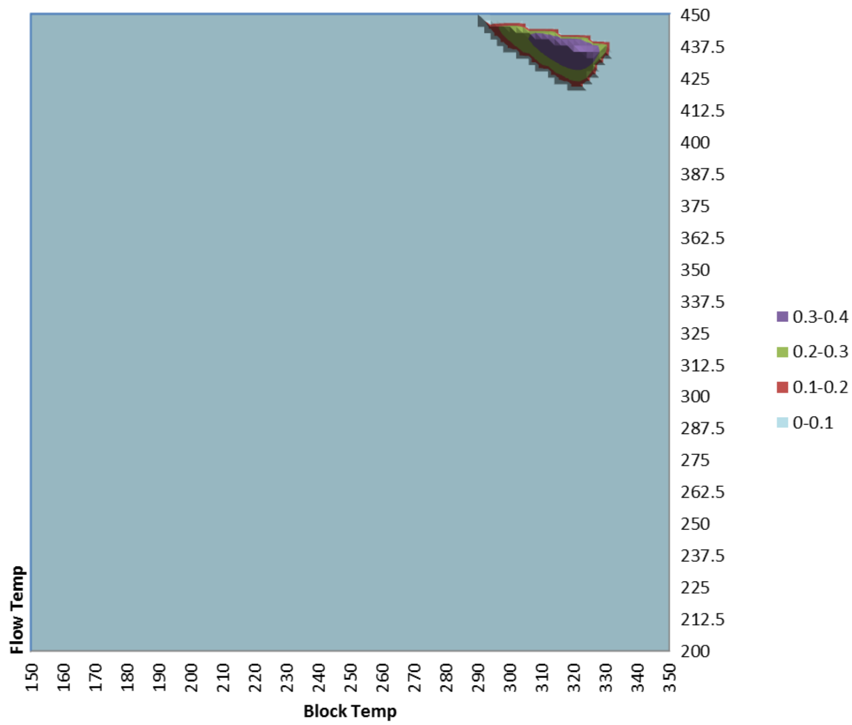


**Figure 13. Value of D as Block Temp and Flow Temp vary (Flow Rate = 120)**

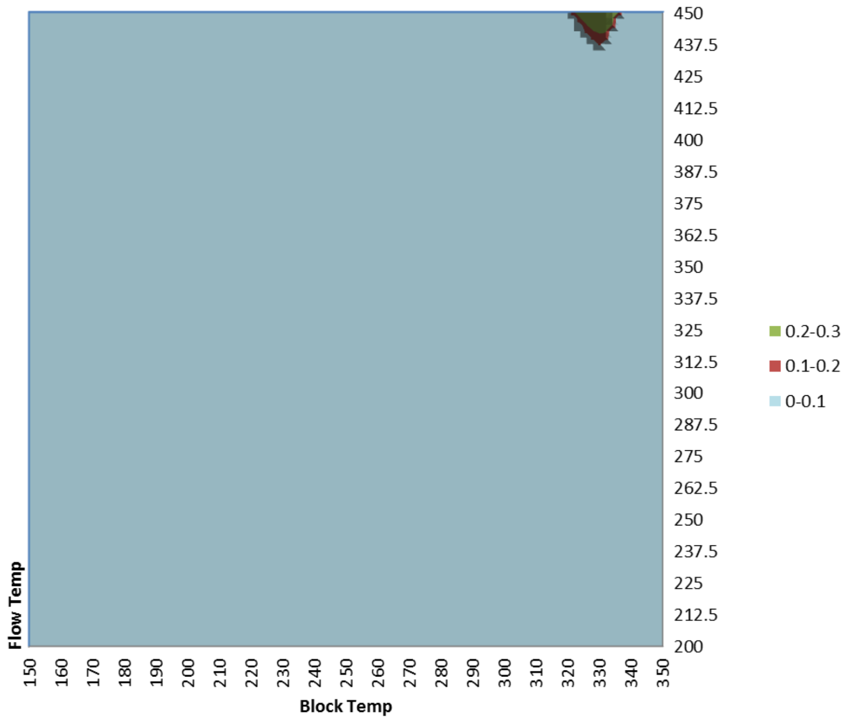


Figure 14. Value of D as Block Temp and Flow Temp vary (Flow Rate = 82.19)

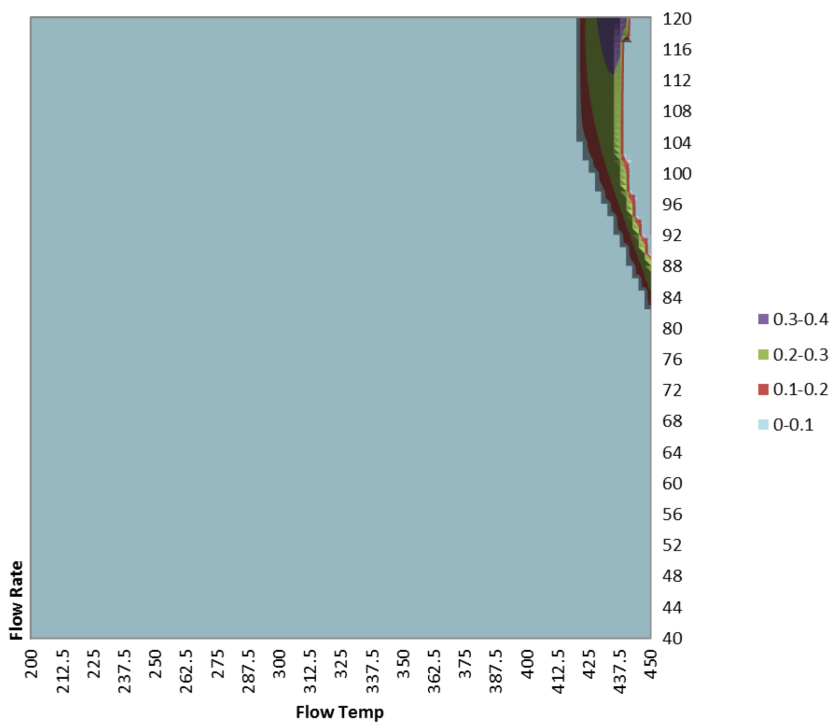


Figure 15. Value of D as Flow Temp and Flow Rate vary (Block Temp = 320.30)

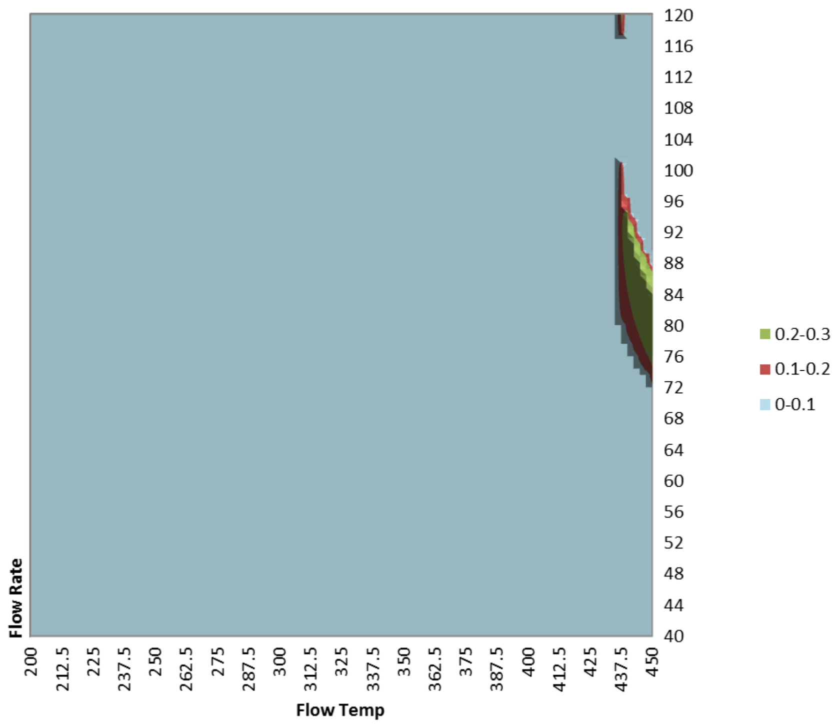


**Figure 16. Value of D as Flow Temp and Flow Rate vary (Block Temp = 330.45)**

# Appendix D. Regression equation coefficients and residuals for NOEM experiment

**Table 34. Coefficients for predicted Debt ($\hat{y}_1$)**

| Factor | Coefficient Estimate |
|---|---|
| Intercept | -1.2E+11 |
| $A$-Adjud Proc Time | -1E+10 |
| $B$-Infra Spend | -9.5E+07 |
| $C$-Govt Employ | -5.2E+10 |
| $D$-Interest Rate | -5.2E+11 |
| $E$-Police Goal | 2223638 |
| $F$-Tax Rate | 1.88E+10 |
| $G$-Adjud Rate | 2.41E+11 |
| $H$-Stimulus Pct | 3.01E+08 |
| $J$-Govt Wage | 34007472 |
| $K$-Service Spend | 52747611 |
| $L$-Corrupt Pct | 5.39E+11 |
| $M$-Police Init | 485203.9 |
| $N$-Jail Term | 1.7E+08 |
| $O$-Stimulus | 2870.185 |
| $AB$ | -1.4E+07 |
| $AC$ | -5.7E+10 |
| $AD$ | 1.21E+10 |
| $AE$ | 34766.79 |
| $AF$ | -5.8E+09 |

Continued on next page.

**Table 34. (Debt model coefficients continued)**

| Factor | Coefficient Estimate |
|---|---|
| $AG$ | -4.4E+10 |
| $AH$ | -3.5E+07 |
| $AJ$ | 292440.8 |
| $AK$ | 30246284 |
| $AL$ | 1.04E+09 |
| $AM$ | 9606.421 |
| $AN$ | -2E+07 |
| $AO$ | 276.1704 |
| $BC$ | -5.2E+08 |
| $BD$ | -7.7E+08 |
| $BE$ | 300.0216 |
| $BF$ | 35535699 |
| $BG$ | -6.3E+07 |
| $BH$ | -205478 |
| $BJ$ | -12612 |
| $BK$ | -124684 |
| $BL$ | -1.3E+08 |
| $BM$ | 277.5306 |
| $BN$ | 91005.42 |
| $BO$ | -1.04642 |
| $CD$ | -1.2E+12 |
| $CE$ | 578785.7 |

Continued on next page.

**Table 34. (Debt model coefficients continued)**

| Factor | Coefficient Estimate |
|---|---|
| $CF$ | 1.22E+11 |
| $CG$ | 6.9E+10 |
| $CH$ | -5.7E+08 |
| $CJ$ | 20315991 |
| $CK$ | -9.3E+08 |
| $CL$ | -1.1E+12 |
| $CM$ | 894714.9 |
| $CN$ | 9.66E+08 |
| $CO$ | 14794.99 |
| $DE$ | -581941 |
| $DF$ | 3.29E+10 |
| $DG$ | -7.8E+11 |
| $DH$ | -8E+08 |
| $DJ$ | 15254317 |
| $DK$ | -7.7E+08 |
| $DL$ | -3.3E+11 |
| $DM$ | 520396.9 |
| $DN$ | 1.44E+08 |
| $DO$ | 5747.516 |
| $EF$ | -61603 |
| $EG$ | -286008 |
| $EH$ | -255.663 |

Continued on next page.

**Table 34. (Debt model coefficients continued)**

| Factor | Coefficient Estimate |
|---|---|
| $EJ$ | 9.591571 |
| $EK$ | 267.4753 |
| $EL$ | -74454 |
| $EM$ | -0.08278 |
| $EN$ | -231.426 |
| $EO$ | 0.003944 |
| $FG$ | -5.3E+10 |
| $FH$ | -3.5E+07 |
| $FJ$ | -495916 |
| $FK$ | 1.17E+08 |
| $FL$ | -7.5E+10 |
| $FM$ | 10285.56 |
| $FN$ | 22431989 |
| $FO$ | 1334.28 |
| $GH$ | 1.35E+08 |
| $GJ$ | -7219620 |
| $GK$ | -1.8E+08 |
| $GL$ | 7.95E+10 |
| $GM$ | 176310 |
| $GN$ | -5222336 |
| $GO$ | 2426.442 |
| $HJ$ | -14446.1 |

Continued on next page.

**Table 34. (Debt model coefficients continued)**

| Factor | Coefficient Estimate |
|---|---|
| $HK$ | -177820 |
| $HL$ | -6.3E+08 |
| $HM$ | 287.7633 |
| $HN$ | -365415 |
| $HO$ | 13.19157 |
| $JK$ | 2268.709 |
| $JL$ | -6658941 |
| $JM$ | -2.26307 |
| $JN$ | -1152.77 |
| $JO$ | 0.126152 |
| $KL$ | -2E+08 |
| $KM$ | 190.0535 |
| $KN$ | 65779.41 |
| $KO$ | -5.14335 |
| $LM$ | -129731 |
| $LN$ | -6E+07 |
| $LO$ | -2756.72 |
| $MN$ | 285.4206 |
| $MO$ | -0.00252 |
| $NO$ | 3.376809 |
| $A^2$ | 3.2E+10 |
| $B^2$ | 3978377 |

Continued on next page.

**Table 34. (Debt model coefficients continued)**

| Factor | Coefficient Estimate |
|---|---|
| $C^2$ | 2.2E+11 |
| $D^2$ | 2.02E+13 |
| $E^2$ | -16.731 |
| $F^2$ | -1.4E+11 |
| $G^2$ | -2.5E+12 |
| $H^2$ | -285702 |
| $J^2$ | -12046.9 |
| $K^2$ | -828766 |
| $L^2$ | -5.4E+12 |
| $M^2$ | -5.3217 |
| $N^2$ | -4390076 |
| $O^2$ | -0.00124 |
| $A^3$ | -2.6E+10 |
| $B^3$ | -27719.5 |
| $C^3$ | 3.87E+12 |
| $D^3$ | -1.6E+14 |
| $E^3$ | 4.06E-05 |
| $F^3$ | 2.04E+11 |
| $G^3$ | 1.2E+13 |
| $H^3$ | -6288.01 |
| $J^3$ | 1.29533 |
| $K^3$ | 6608.289 |

Continued on next page.

**Table 34. (Debt model coefficients continued)**

| **Factor** | **Coefficient Estimate** |
|---|---|
| $L^3$ | 1.95E+13 |
| $M^3$ | 1.51E-05 |
| $N^3$ | 26475.31 |
| $O^3$ | 8.38E-11 |

**Table 35. Coefficients for predicted activists** $(\ln \hat{y}_2)$

| **Factor** | **Coefficient Estimate** |
|---|---|
| Intercept | 39.655 |
| $A$-Adjud Proc Time | 1.762 |
| $B$-Infra Spend | 0.054 |
| $C$-Govt Employ | -141.256 |
| $D$-Interest Rate | -97.080 |
| $E$-Police Goal | -0.001 |
| $F$-Tax Rate | 3.105 |
| $G$-Adjud Rate | -73.038 |
| $H$-Stimulus Pct | -0.020 |
| $J$-Govt Wage | -0.002 |
| $K$-Service Spend | -0.009 |
| $L$-Corrupt Pct | 320.777 |
| $M$-Police Init | 9.50E-06 |

Continued on next page.

**Table 35. (Activist model coefficients continued)**

| Factor | Coefficient Estimate |
|---|---|
| $N$-Jail Term | -0.091 |
| $O$-Stimulus | -6.10E-07 |
| $AB$ | -0.002 |
| $AC$ | 12.913 |
| $AD$ | -1.894 |
| $AE$ | -7.71E-06 |
| $AF$ | 0.151 |
| $AG$ | -4.845 |
| $AH$ | 0.002 |
| $AJ$ | 8.75E-05 |
| $AK$ | 0.007 |
| $AL$ | 1.518 |
| $AM$ | 5.64E-06 |
| $AN$ | 0.003 |
| $AO$ | -2.59E-08 |
| $BC$ | 0.039 |
| $BD$ | 0.035 |
| $BE$ | 7.29E-09 |
| $BF$ | -0.005 |
| $BG$ | 0.029 |
| $BH$ | -3.30E-06 |
| $BJ$ | 1.39E-07 |

Continued on next page.

**Table 35. (Activist model coefficients continued)**

| Factor | Coefficient Estimate |
|---|---|
| $BK$ | 1.39E-06 |
| $BL$ | 0.013 |
| $BM$ | -4.98E-08 |
| $BN$ | -6.05E-06 |
| $BO$ | -1.71E-10 |
| $CD$ | -168.524 |
| $CE$ | -1.44E-04 |
| $CF$ | -16.719 |
| $CG$ | 46.315 |
| $CH$ | 0.065 |
| $CJ$ | -0.002 |
| $CK$ | 0.106 |
| $CL$ | 192.671 |
| $CM$ | 6.72E-05 |
| $CN$ | 0.054 |
| $CO$ | -8.55E-07 |
| $DE$ | 9.77E-05 |
| $DF$ | -34.473 |
| $DG$ | -81.507 |
| $DH$ | -0.030 |
| $DJ$ | 0.008 |
| $DK$ | -0.067 |

Continued on next page.

**Table 35. (Activist model coefficients continued)**

| Factor | Coefficient Estimate |
|---|---|
| $DL$ | -60.734 |
| $DM$ | -4.83E-05 |
| $DN$ | -0.001 |
| $DO$ | -9.20E-07 |
| $EF$ | 1.98E-05 |
| $EG$ | 2.05E-05 |
| $EH$ | -5.40E-08 |
| $EJ$ | -7.05E-11 |
| $EK$ | -5.09E-08 |
| $EL$ | 5.38E-05 |
| $EM$ | 7.64E-11 |
| $EN$ | -6.25E-08 |
| $EO$ | 5.96E-13 |
| $FG$ | -0.475 |
| $FH$ | 0.006 |
| $FJ$ | -0.001 |
| $FK$ | 0.002 |
| $FL$ | -0.111 |
| $FM$ | 7.26E-06 |
| $FN$ | -0.008 |
| $FO$ | -6.60E-09 |
| $GH$ | -0.026 |

Continued on next page.

**Table 35. (Activist model coefficients continued)**

| Factor | Coefficient Estimate |
|---|---|
| $GJ$ | -0.001 |
| $GK$ | 0.013 |
| $GL$ | 0.190 |
| $GM$ | 2.37E-05 |
| $GN$ | 0.024 |
| $GO$ | 4.19E-08 |
| $HJ$ | -1.26E-06 |
| $HK$ | 1.38E-05 |
| $HL$ | 0.046 |
| $HM$ | 4.55E-08 |
| $HN$ | 5.80E-05 |
| $HO$ | 5.04E-10 |
| $JK$ | -1.34E-06 |
| $JL$ | -3.17E-04 |
| $JM$ | 6.59E-10 |
| $JN$ | 9.41E-07 |
| $JO$ | 8.55E-13 |
| $KL$ | 0.076 |
| $KM$ | -4.62E-08 |
| $KN$ | 6.84E-05 |
| $KO$ | 1.08E-10 |
| $LM$ | 5.68E-05 |

Continued on next page.

**Table 35. (Activist model coefficients continued)**

| Factor | Coefficient Estimate |
|---|---|
| $LN$ | 0.058 |
| $LO$ | 7.64E-07 |
| $MN$ | -1.84E-08 |
| $MO$ | -8.70E-13 |
| $NO$ | 7.54E-10 |
| $A^2$ | -7.008 |
| $B^2$ | -0.001 |
| $C^2$ | 7228.526 |
| $D^2$ | 2707.111 |
| $E^2$ | 4.72E-09 |
| $F^2$ | -10.416 |
| $G^2$ | 1169.703 |
| $H^2$ | 8.40E-05 |
| $J^2$ | 7.16E-07 |
| $K^2$ | 5.42E-05 |
| $L^2$ | -2686.560 |
| $M^2$ | -1.95E-10 |
| $N^2$ | 0.001 |
| $O^2$ | 1.84E-13 |
| $A^3$ | 5.351 |
| $B^3$ | 6.73E-06 |
| $C^3$ | -94855.894 |

Continued on next page.

**Table 35. (Activist model coefficients continued)**

| Factor | Coefficient Estimate |
|---|---|
| $D^3$ | -21722.951 |
| $E^3$ | -9.94E-15 |
| $F^3$ | 11.252 |
| $G^3$ | -4959.153 |
| $H^3$ | 5.56E-07 |
| $J^3$ | -7.44E-11 |
| $K^3$ | -2.09E-07 |
| $L^3$ | 7234.906 |
| $M^3$ | 1.87E-16 |
| $N^3$ | -5.68E-06 |
| $O^3$ | -1.70E-20 |

**Table 36. Residuals from NOEM experiment**

| n | Residual $y_1$ | Residual ln $\boldsymbol{y_2}$ |
|---|---|---|
| 1 | 6.05E+08 | 0.01470 |
| 2 | 46501523 | 0.04140 |
| 3 | -1.4E+08 | -2.26903 |
| 4 | -5.2E+08 | 0.02669 |
| 5 | 17163091 | 0.00104 |
| 6 | 2.93E+08 | 0.00221 |

Continued on next page.

Table 36. (Residuals continued)

| n | Residual $y_1$ | Residual $\ln\ y_2$ |
|---|---|---|
| 7 | 6.07E+08 | -0.03120 |
| 8 | -2.1E+08 | 0.01059 |
| 9 | -6.3E+07 | -0.00359 |
| 10 | -5.9E+08 | -0.00723 |
| 11 | -8.6E+08 | -0.03596 |
| 12 | 5.91E+08 | 0.12739 |
| 13 | 3.27E+08 | -0.00834 |
| 14 | -4.7E+08 | 0.05884 |
| 15 | -8.1E+08 | 0.04631 |
| 16 | 78070523 | -0.00677 |
| 17 | 4.12E+08 | 0.02850 |
| 18 | 1.1E+08 | 0.00031 |
| 19 | 6.28E+08 | -0.09888 |
| 20 | 3.9E+08 | -0.03841 |
| 21 | 7.14E+08 | -0.22464 |
| 22 | -3.1E+08 | -0.11321 |
| 23 | -3E+08 | 0.01299 |
| 24 | 3.59E+08 | -0.02980 |
| 25 | -5.1E+08 | 0.05644 |
| 26 | -8.7E+08 | 0.06156 |
| 27 | 5.54E+08 | 0.14357 |
| 28 | -1.7E+08 | 0.01179 |

Continued on next page.

Table 36. (Residuals continued)

| n | Residual $y_1$ | Residual $\ln\ y_2$ |
|---|---|---|
| 29 | 1.57E+08 | 0.00506 |
| 30 | -6.1E+08 | 0.19792 |
| 31 | 60858431 | -0.07986 |
| 32 | -6.9E+08 | -0.09111 |
| 33 | 4.65E+08 | -0.16477 |
| 34 | 2.89E+08 | -0.00784 |
| 35 | 2.45E+08 | -0.02627 |
| 36 | 1.06E+08 | 0.00367 |
| 37 | 4.94E+08 | -0.03780 |
| 38 | -1.6E+07 | -0.09847 |
| 39 | -2.6E+08 | 0.11083 |
| 40 | -1.9E+08 | 0.16481 |
| 41 | -8.9E+07 | -0.01404 |
| 42 | -3.4E+08 | 0.02320 |
| 43 | -5.5E+08 | -0.03663 |
| 44 | -2.5E+08 | 0.09646 |
| 45 | 14818895 | -0.05011 |
| 46 | -2.2E+08 | -0.02335 |
| 47 | 8.54E+08 | 0.09151 |
| 48 | 1.62E+08 | -0.00864 |
| 49 | -1.4E+08 | 0.00599 |
| 50 | 1.95E+08 | -0.03013 |

Continued on next page.

Table 36. (Residuals continued)

| n | Residual $y_1$ | Residual $\ln\ y_2$ |
|---|---|---|
| 51 | -6E+08 | -0.06399 |
| 52 | 7.77E+08 | -0.06574 |
| 53 | -3.6E+08 | 0.00710 |
| 54 | -7.7E+08 | 0.05698 |
| 55 | -68764.7 | 0.00169 |
| 56 | -6.6E+08 | 0.04321 |
| 57 | 1.07E+08 | -0.03385 |
| 58 | -2.7E+08 | 0.01978 |
| 59 | 1.89E+09 | -0.14355 |
| 60 | -4.5E+08 | 0.03494 |
| 61 | -5.3E+08 | 0.00734 |
| 62 | 2.85E+08 | -0.04153 |
| 63 | 6.09E+08 | -0.03008 |
| 64 | -4.2E+08 | 0.01458 |
| 65 | 1E+09 | -0.14547 |
| 66 | -4.4E+09 | 0.23652 |
| 67 | -3.9E+08 | 0.02111 |
| 68 | 1.42E+08 | -0.04746 |
| 69 | 7.52E+08 | -0.01527 |
| 70 | 4.48E+08 | -0.14286 |
| 71 | 5.77E+08 | -0.07107 |
| 72 | 2424187 | 0.00200 |

Continued on next page.

**Table 36. (Residuals continued)**

| n | Residual $y_1$ | Residual ln $\boldsymbol{y_2}$ |
|---|---|---|
| 73 | -3.2E+07 | 0.00180 |
| 74 | 2.4E+08 | -0.01881 |
| 75 | -2.6E+07 | -0.01237 |
| 76 | 1.91E+08 | 0.24609 |
| 77 | 1.26E+09 | -0.07518 |
| 78 | -5E+08 | 0.04178 |
| 79 | -1.3E+08 | 0.05549 |
| 80 | 7.76E+08 | -0.03767 |
| 81 | 2.86E+08 | 0.08801 |
| 82 | 8.21E+08 | -0.07016 |
| 83 | -2.2E+08 | -0.00163 |
| 84 | 3.11E+08 | -0.04712 |
| 85 | 2.03E+08 | -0.00505 |
| 86 | -3.6E+08 | 0.01480 |
| 87 | 4.66E+08 | -0.09495 |
| 88 | -3.8E+08 | 0.09892 |
| 89 | 5.65E+08 | -0.12924 |
| 90 | -2.3E+08 | 0.01413 |
| 91 | -7.2E+07 | 0.00131 |
| 92 | 1.1E+09 | -0.06148 |
| 93 | -7.7E+08 | 0.04685 |
| 94 | 1.87E+08 | -0.01776 |

Continued on next page.

Table 36. **(Residuals continued)**

| n | Residual $y_1$ | Residual ln $\boldsymbol{y_2}$ |
|---|---|---|
| 95 | 24635434 | 0.00560 |
| 96 | -5.5E+08 | 0.03593 |
| 97 | -2.3E+08 | 0.02691 |
| 98 | -4.1E+08 | 0.01791 |
| 99 | -5.2E+08 | 0.03291 |
| 100 | 1.09E+09 | -0.06534 |
| 101 | -2.7E+07 | 0.17692 |
| 102 | -6.9E+08 | 0.13842 |
| 103 | -1.1E+08 | 0.10242 |
| 104 | -1.3E+07 | -0.09554 |
| 105 | 74725502 | -0.01007 |
| 106 | -9E+08 | 0.05072 |
| 107 | -8.6E+07 | 0.00804 |
| 108 | 7.68E+08 | -0.04576 |
| 109 | 77373775 | -0.00131 |
| 110 | -6.2E+08 | 0.08858 |
| 111 | 4.61E+08 | -0.02004 |
| 112 | 2.87E+08 | -0.03326 |
| 113 | 5.75E+08 | -0.23979 |
| 114 | -4.3E+08 | 0.01492 |
| 115 | 20782539 | 0.09571 |
| 116 | 20906907 | -0.07693 |

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Table 36. (Residuals continued)

| n | Residual $y_1$ | Residual $\ln\ y_2$ |
|---|---|---|
| 117 | 6.44E+08 | -0.02522 |
| 118 | 3.28E+08 | -0.01519 |
| 119 | 1.39E+08 | 0.11427 |
| 120 | -2.3E+08 | -0.06090 |
| 121 | 1.02E+09 | 0.04245 |
| 122 | -7E+08 | 0.14105 |
| 123 | -1.6E+08 | -0.00421 |
| 124 | 21789422 | 0.00127 |
| 125 | -6.1E+08 | 0.03328 |
| 126 | -2.2E+08 | -0.00190 |
| 127 | 88528846 | -0.00964 |
| 128 | 82073456 | -0.02887 |
| 129 | -5.4E+08 | 0.00532 |
| 130 | -7.4E+08 | 0.04756 |
| 131 | 88697026 | -0.01250 |
| 132 | -7.8E+08 | 0.07467 |
| 133 | -9.4E+07 | -0.18327 |
| 134 | -5.2E+08 | -0.00225 |
| 135 | 5.25E+08 | 0.00848 |
| 136 | 3.65E+08 | 0.00283 |
| 137 | 3.14E+08 | -0.04048 |
| 138 | 6.33E+08 | -0.04005 |

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**Table 36. (Residuals continued)**

| n | Residual $y_1$ | Residual $\ln\ y_2$ |
|---|---|---|
| 139 | 2.58E+08 | -0.00268 |
| 140 | 6.45E+08 | -0.09468 |
| 141 | 4.89E+08 | -0.06711 |
| 142 | -3.1E+08 | 2.29606 |
| 143 | -5.2E+08 | 0.02387 |
| 144 | 11918414 | -0.00055 |
| 145 | 2.85E+08 | -0.04482 |
| 146 | 6.06E+08 | -0.03489 |
| 147 | -2E+08 | 0.01573 |
| 148 | -7.4E+07 | 0.01549 |
| 149 | -5.9E+08 | 0.07362 |
| 150 | 63301594 | 0.08141 |
| 151 | 8.05E+08 | -0.21154 |
| 152 | 3.28E+08 | -0.00924 |
| 153 | -5.3E+08 | -0.02777 |
| 154 | -8.1E+08 | 0.04623 |
| 155 | 84505897 | -0.00937 |
| 156 | 4.07E+08 | -0.07991 |
| 157 | 1.06E+08 | -0.01242 |
| 158 | 6.27E+08 | 0.04279 |
| 159 | 3.99E+08 | 0.01167 |
| 160 | 6.79E+08 | 0.15372 |

Continued on next page.

**Table 36. (Residuals continued)**

| n | Residual $y_1$ | Residual ln $y_2$ |
|---|---|---|
| 161 | -3.5E+08 | 0.13814 |
| 162 | -3E+08 | 0.02363 |
| 163 | 3.6E+08 | -0.01006 |
| 164 | -5.2E+08 | 0.01254 |
| 165 | -8.8E+08 | 0.04320 |
| 166 | 5.77E+08 | -0.19703 |
| 167 | -2.2E+08 | 0.01890 |
| 168 | 1.68E+08 | -0.03330 |
| 169 | -5.9E+08 | -0.12573 |
| 170 | 64175139 | 0.07030 |
| 171 | -6.9E+08 | 0.17001 |
| 172 | 4.65E+08 | 0.11906 |
| 173 | 3.02E+08 | -0.02906 |
| 174 | 2.46E+08 | 0.00041 |
| 175 | 1.09E+08 | -0.01239 |
| 176 | 6.07E+08 | -0.02233 |
| 177 | -2.3E+07 | -0.11000 |
| 178 | -2.6E+08 | 0.12963 |
| 179 | -1.9E+08 | -0.15507 |
| 180 | -8.9E+07 | 0.01462 |
| 181 | -3.5E+08 | 0.00679 |
| 182 | -5.5E+08 | 0.10022 |

Continued on next page.

**Table 36. (Residuals continued)**

| n | Residual $y_1$ | Residual $\ln\ y_2$ |
|---|---|---|
| 183 | -2.6E+08 | -0.05576 |
| 184 | 14569111 | 0.05139 |
| 185 | -2.2E+08 | 0.04875 |
| 186 | 1.03E+09 | -0.20512 |
| 187 | 1.98E+08 | -0.01911 |
| 188 | -1.3E+08 | 0.00501 |
| 189 | 1.86E+08 | 0.00298 |
| 190 | -1E+09 | 0.16006 |
| 191 | 6.57E+08 | -0.02194 |
| 192 | -1.6E+08 | 0.01964 |
| 193 | -7.7E+08 | 0.03657 |
| 194 | -1561419 | 0.00145 |
| 195 | -4.4E+08 | 0.02101 |
| 196 | 1.2E+08 | 0.01994 |
| 197 | -3E+08 | 0.01753 |
| 198 | 1.97E+09 | -0.13089 |
| 199 | -4.5E+08 | 0.01859 |
| 200 | -6E+08 | 0.06112 |
| 201 | 1.21E+08 | 0.03122 |
| 202 | 5.93E+08 | -0.02883 |
| 203 | -4.9E+08 | 0.04262 |
| 204 | 9.95E+08 | 0.04702 |

Continued on next page.

Table 36. (Residuals continued)

| n | Residual $y_1$ | Residual $\ln\ y_2$ |
|---|---|---|
| 205 | -4.4E+09 | 0.23771 |
| 206 | -3.6E+08 | 0.02866 |
| 207 | 1.85E+08 | 0.04997 |
| 208 | 7.1E+08 | -0.06078 |
| 209 | 4.4E+08 | 0.08851 |
| 210 | 5.66E+08 | 0.00929 |
| 211 | -1.7E+07 | 0.00199 |
| 212 | -7.9E+07 | 0.00215 |
| 213 | 2.58E+08 | -0.01779 |
| 214 | -3E+07 | -0.00057 |
| 215 | 1.86E+08 | -0.27432 |
| 216 | 1.28E+09 | -0.07517 |
| 217 | -5.4E+08 | 0.02574 |
| 218 | 37521805 | -0.05756 |
| 219 | 7.48E+08 | -0.03814 |
| 220 | 2.94E+08 | -0.10614 |
| 221 | 8.27E+08 | -0.02067 |
| 222 | -2.2E+08 | 0.02649 |
| 223 | 3.09E+08 | -0.00408 |
| 224 | 1.92E+08 | -0.00951 |
| 225 | -2.7E+08 | 0.01268 |
| 226 | 4.97E+08 | 0.04459 |

Continued on next page.

**Table 36. (Residuals continued)**

| n | Residual $y_1$ | Residual $\ln\ y_2$ |
|---|---|---|
| 227 | -3.2E+08 | -0.04864 |
| 228 | 59462511 | 0.09071 |
| 229 | -6.5E+07 | -0.01050 |
| 230 | -1.4E+08 | 0.01054 |
| 231 | 1.14E+09 | -0.06252 |
| 232 | -8.5E+08 | 0.04684 |
| 233 | 1.94E+08 | 0.00299 |
| 234 | -1.2E+08 | -0.00682 |
| 235 | -5.5E+08 | 0.01613 |
| 236 | -2.8E+08 | -0.00165 |
| 237 | -5.5E+08 | 0.04358 |
| 238 | -5.1E+08 | 0.03270 |
| 239 | 1.09E+09 | -0.06537 |
| 240 | 5.74E+08 | -0.20988 |
| 241 | -6.9E+08 | -0.06992 |
| 242 | -5.7E+07 | -0.08715 |
| 243 | -6.1E+08 | 0.13442 |
| 244 | 1.34E+08 | -0.00347 |
| 245 | -9E+08 | 0.05345 |
| 246 | -8.4E+07 | 0.00772 |
| 247 | 7.62E+08 | -0.04690 |
| 248 | 891461.7 | -0.00482 |

Continued on next page.

**Table 36. (Residuals continued)**

| n | Residual $y_1$ | Residual $\ln\ \boldsymbol{y_2}$ |
|---|---|---|
| 249 | -6.5E+08 | -0.01392 |
| 250 | 3.75E+08 | -0.02906 |
| 251 | 2.66E+08 | 0.00046 |
| 252 | 5.86E+08 | 0.17459 |
| 253 | -4.3E+08 | 0.03501 |
| 254 | 20776848 | 0.05542 |
| 255 | 20998168 | -0.07753 |
| 256 | 6.51E+08 | -0.03974 |
| 257 | 3.28E+08 | -0.00574 |
| 258 | 1.77E+08 | -0.12397 |
| 259 | -2.4E+08 | 0.08025 |
| 260 | 1.06E+09 | -0.15035 |
| 261 | -6.4E+08 | -0.05990 |
| 262 | -2.5E+08 | 0.02545 |
| 263 | 16457326 | -0.00119 |
| 264 | -5.5E+08 | 0.02066 |
| 265 | -2.3E+08 | 0.04429 |
| 266 | -6.4E+07 | -0.00435 |
| 267 | 1.44E+08 | 0.01031 |
| 268 | -5.4E+08 | 0.06502 |
| 269 | -7.4E+08 | 0.02541 |
| 270 | 82312932 | 0.00280 |

Continued on next page.

**Table 36. (Residuals continued)**

| n | Residual $y_1$ | Residual ln $\boldsymbol{y_2}$ |
|---|---|---|
| 271 | -7.8E+08 | 0.01035 |
| 272 | -2.1E+08 | 0.19584 |
| 273 | -5.2E+08 | 0.06252 |
| 274 | 5.24E+08 | -0.07190 |
| 275 | 3.56E+08 | -0.03884 |
| 276 | 3.34E+08 | -0.00970 |
| 277 | 6.4E+08 | -0.03549 |
| 278 | 2.06E+08 | -0.01601 |

# Appendix E. Quad Chart

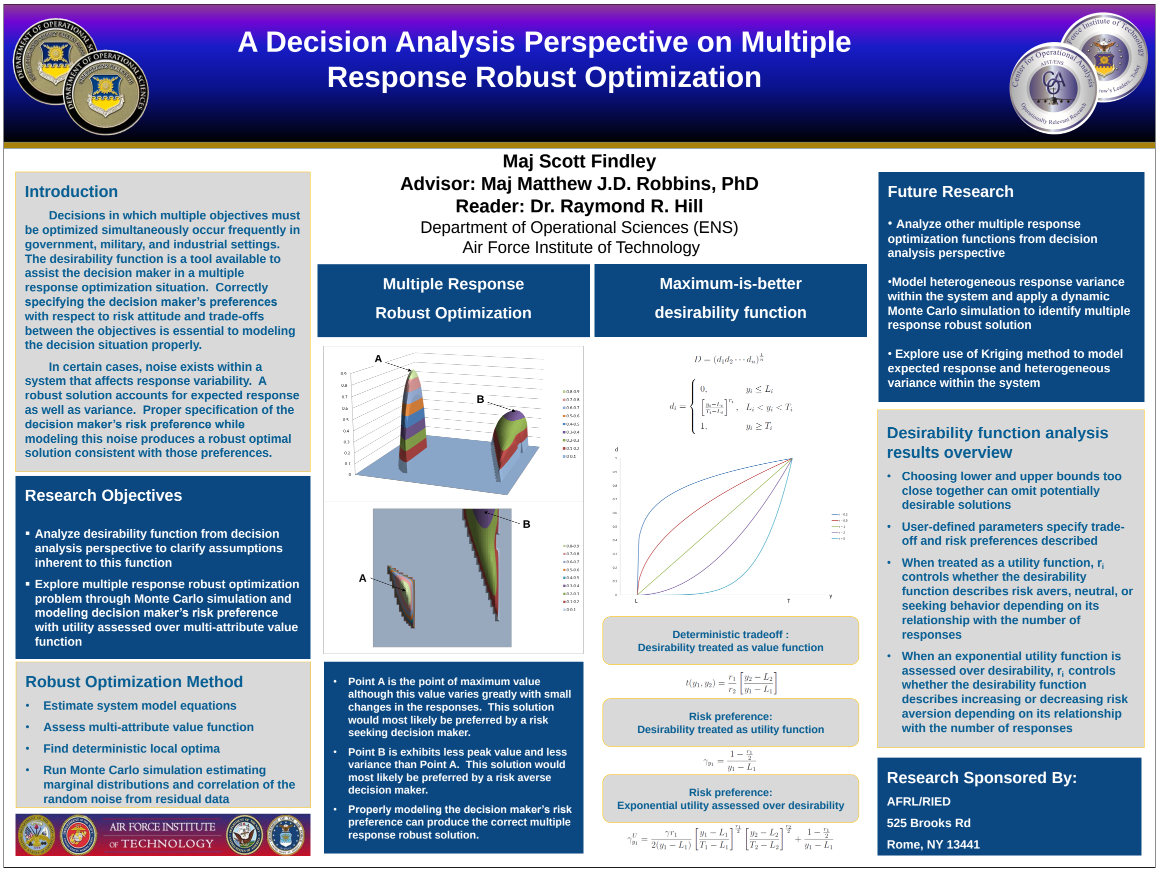


Figure 17. Quad Chart

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Decisions in which multiple objectives must be optimized simultaneously occur frequently in government, military, and industrial settings. One method a decision maker may use to assist in such decisions is the application of a desirability function. An informed specification of the desirability function's parameters is essential to accurately describe the decision maker's value trade-offs and risk preference. This thesis uses utility transversality to analyze the implicit trade-off and risk attitude assumptions attendant to the desirability function.

The desirability function does not explicitly account for response variability. A robust solution considers not only the expected response into account but also its variance. Assessing a utility function over desirability as a means to describe the decision maker's risk attitude produces a robust operating solution consistent with those preferences. This thesis examines robustness as it applies to the desirability function in a manufacturing experiment example.

Different levels of diplomatic, informational, military and economic (DIME) instruments of national policy are investigated to examine their effect on the political, military, economic, social, infrastructure, and information (PMESII) systems of a nation, AFRL's National Operational Environment Model (NOEM) serves as a basis for identifying a robust national policy in a scenario involving the Democratic Republic of Congo.

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